International Journal of Quantum Information
Vol. 5, Nos. 1 & 2 (2007) 235–240

EXPERIMENTAL ANALYSIS OF A SIMPLE LINEAR OPTICS PHASE GATE

CH. SCHMID[*,†,§], N. KIESEL[*,†], U. WEBER[*,†], R. URSIN[‡]
and H. WEINFURTER[*,†]

[*] *Max-Planck-Institute of Quantum Optics, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*

[†] *Department of Physics, Ludwig-Maximilians-University, Schellingstr. 4/III, D-80799 Munich, Germany*

[‡] *Institute for Experimental Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria*

[§] *christian.schmid@mpq.mpg.de*

Linear optics two-qubit logic gates are essential tools in photonic quantum information. We describe a recently introduced simple conditional phase gate for photons, which relies on only one second order interference at a polarization dependent beam splitter, thereby making additional stability precautions dispensable. The improved quality of the gate is evaluated by performing full process tomography. The obtained process tomography data is fitted by a model based on experimental parameters of the setup which allows predictions on the performance of the gate in multi-photon experiments.

Keywords: Linear optics two-qubit gate; cluster state; entanglement swapping.

It was shown for qubits that entangling gates like the controlled phase (C-phase) or controlled NOT (CNOT) gate together with single qubit operations are sufficient to create any kind of quantum network. From that point of view these gates are essential tools in quantum information science. The quantum system of choice for the implementation of a lot of quantum information tasks are photons, as their interaction with the environment is small guaranteeing low decoherence. While the creation of entangled photon pairs via spontaneous parametric down conversion (SPDC) became a standard technique, the realization of two photon quantum gates is still a major challenge, mainly due to low nonlinear interaction efficiencies. One solution to this problem is — as long as one focuses on performing only a limited number of quantum logic operations — to use linear optics components and conditioned detection. In this case the action of the operation is restricted to the detection of one photon in each output port of the gate, which will occur only with a certain probability. Recently we introduced a linear optics C-Phase gate,[1] which uses a single two-photon interference at a polarization dependent beam splitter. The C-phase action is obtained with probability 1/9 and the stability requirements are relaxed to the coherence length of the detected photons ($\approx 150\,\mu$m), what can easily be fulfilled without additional stabilization equipment. Therefore the gate looks promising with respect to application in multi-photon experiments. To characterize the gate we used linear quantum process tomography (QPT).[2–4] The obtained process matrix was fitted by a theoretical model based on experimental parameters of the setup. In the following we recapitulate the functionality of the gate and study how well the implemented gate is described by our derived model. On account of this we simulate the creation of a four photon cluster state by using the C-phase gate model and compare the results with actual experimental data. Finally we discuss the expected performance of the gate for complete Bell state analysis in an entanglement swapping experiment.

The ideal C-Phase gate acts on two-qubit input states $|\psi_{\text{in}}\rangle = (c_{HH}|HH\rangle + c_{HV}|HV\rangle + c_{VH}|VH\rangle + c_{VV}|VV\rangle)$, and applies a relative π-phase shift to the contribution $|VV\rangle$ only, such that $|\psi_{\text{out}}\rangle = (c_{HH}|HH\rangle + c_{HV}|HV\rangle + c_{VH}|VH\rangle - c_{VV}|VV\rangle)$. Here we encode the logical 0 (1) in linear horizontal H (vertical V) polarization of photons. c_{HH} denotes the amplitude of the $|HH\rangle$-term (for the other terms accordingly).

In our implementation the application of the phase shift relies on second-order interference of indistinguishable photons at a polarization dependent beam splitter (PDBS)[a] (Fig. 1).[5,6] The transmission of 1/3 for vertical polarization results in a total amplitude of $-1/3$ for the $|VV\rangle$ output terms, as can be seen by adding

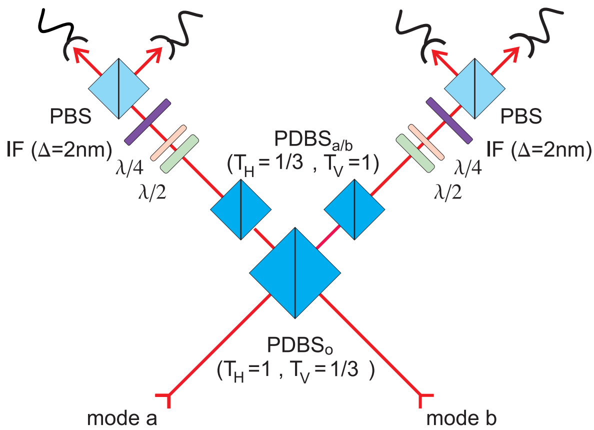


Fig. 1. Experimental setup for the C-Phase gate. The phase is introduced by a second order interference at a polarization dependent beam splitter PDBS_O. To obtain equal output amplitudes for any input polarization state polarization dependent beam splitters with reversed splitting ratio $\text{PDBS}_{a/b}$ are placed in each mode. The gate operation is applied in case of a coincidence detection between modes a and b. The resulting output state is analyzed via half- and quarter-wave plates $\lambda/2$, $\lambda/4$ and a polarizing beam splitter PBS.

[a]For each PDBS, we use a custom made beam splitter with dielectric coating designed for the purpose. The PDBS_O is realized as a beam splitting cube, while $\text{PDBS}_{a/b}$ are plates.

the amplitudes for a coincident detection: $(t_V^a \cdot t_V^b) + (ir_V^a \cdot ir_V^b) = \sqrt{1/3}\sqrt{1/3} - \sqrt{2/3}\sqrt{2/3} = -1/3$ where t_i^x (r_i^x) is the amplitude for transmission (reflection) of state $|i\rangle$ in mode x. Perfect transmission of horizontal polarization causes that no interference happens on the contributions $|HH\rangle, |HV\rangle$ and $|VH\rangle$. As the absolute values of the amplitudes need to be equal for any input we still need to attenuate the contributions that include horizontal polarization. This is achieved by $\mathrm{PDBS_{a/b}}$ with the transmission 1/3 for horizontal polarization and perfect transmission for vertical polarization in both output modes. All together we find a probability of 1/9 to obtain a coincidence in the outputs and thus a gate operation with perfect fidelity.

Working with real components results in deviations from the theoretical derivation. A detailed calculation with arbitrary transmission and reflection amplitudes at $\mathrm{PDBS_O}$ and $\mathrm{PDBS_{a,b}}$ shows how their parameters influence the gate operation. To obtain the expected C-phase gate operation one has to fulfill several conditions. First, $(r_V^a r_V^b)/(t_V^a t_V^b) = 2$, which is approximately achieved by slightly varying the angle of incidence at $\mathrm{PDBS_O}$. Experimentally we reach a value of 2.018 ± 0.003. Second, $r_H^a = 0 = r_H^b$, which requires the reflection of the horizontal polarization at the overlap beam splitter to be zero. The third condition, $t_H^a a_H = t_V^a a_V$, and $t_H^b b_H = t_V^b b_V$, respectively, determines the setting for the attenuation at $\mathrm{PDBS}_{a,b}$, where a_i (b_i) are the transmission amplitudes of $|i\rangle$ in mode a (b).

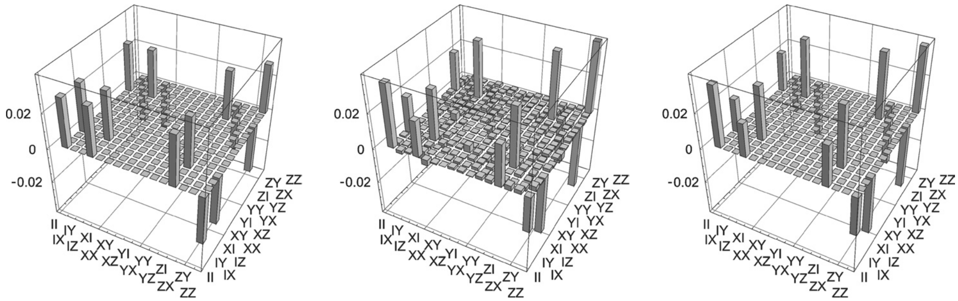
To experimentally test the gate operation we used photon pairs emitted from SPDC. A 2 mm thick BBO (β-Barium Borate) crystal was pumped by UV pump pulses with a central wavelength of 390 nm and an average power of 700 mW from a frequency-doubled mode-locked Ti:sapphire laser (pulse length 130 fs). As the gate is intended to work in multi-photon applications we preferred to characterize it for pulsed mode of operation. The emission is filtered with polarizers to prepare input product states with high quality. Identical spatial modes are guaranteed by coupling the photon pairs into single mode fibres. The spectral mode selection is improved via 2 nm bandwidth filters behind the gate. Information about the indistinguishability of the photons at the $\mathrm{PDBS_O}$ is obtained from a Hong–Ou–Mandel[5] (HOM) dip-measurement. We call the ratio of the experimentally observed and the theoretically expected dip-visibility $\mathcal{V}$ overlap quality $\mathcal{Q} = \mathcal{V}_{\mathrm{exp}}/\mathcal{V}_{\mathrm{th}}$. For $|VV\rangle$-input we achieved $\mathcal{Q} = 91.0\% \pm 0.9\%$.

For a complete characterization of our C-phase process we use quantum process tomography (QPT). In QPT the process is represented by a superoperator $\mathcal{E}$ which is decomposed in a linear combination of a basis of unitary transformations E_i:

$$\mathcal{E}(\rho_{\mathrm{in}}) = \sum_{i,j} \chi_{ij} E_i \rho_{\mathrm{in}} E_j^\dagger. \tag{1}$$

The matrix χ completely describes the process. In order to obtain all components χ_{ij}, the output density matrices ρ_{out}^k for a tomographic set of the 16 separable input states $|k\rangle|l\rangle$ with[b] $k, l \in \{H, V, +, L\}$ are measured.[7] Figure 2(a) shows the process

[b] $|+\rangle = 1/\sqrt{2}(|H\rangle + |V\rangle)$, $|L\rangle = 1/\sqrt{2}(|H\rangle + i|V\rangle)$.



(a) (b) (c)

Fig. 2. (a) χ matrix of the QPT for an ideal phase gate, (b) for the experimentally realized gate and (c) for a theoretical model fit to the experimental data. The imaginary part of the experimentally obtained χ consists of noise only which is comparable to the one in the real part.

matrix χ_{th} of the ideal linear optics phase gate. It represents the decomposition of the C-Phase gate into unitary operations, for our choice of E_i resulting in

$$\overline{PG}_{\text{ideal}} = (\mathbb{1} \otimes \mathbb{1} + \sigma_z \otimes \mathbb{1} + \mathbb{1} \otimes \sigma_z - \sigma_z \otimes \sigma_z)\,/3\,. \tag{2}$$

This matrix can be compared with the experimentally obtained one [Fig. 2(b), imaginary parts are close to zero: average 0.0 ± 0.002]. From the estimated process tomography matrix we calculated a process fidelity of $F_p = \text{Tr}(\chi_{\text{th}}.\chi_{\text{exp}})/(\text{Tr}(\chi_{\text{th}}).\text{Tr}(\chi_{\text{exp}})) = 81.8\%$. Still, due to Poissonian counting statistics χ_{exp} has non-physical, negative eigenvalues. To circumvent this problem and to gain information about the functionality of the real gate, we describe it via a theoretical model including experimental parameters.

The transformation of the phase gate consists of interference between both photons transmitted or both photons reflected $\overline{PG}_{\text{gen}} = M_{tt} + M_{rr}$, where both M_{tt} and M_{rr} are matrices with components given by the transmission-, reflection- and attenuation-amplitudes. For simplicity we assume $t_V^a = t_V^b$ and $r_H^a = r_H^b = 0$. $M_{rr} = |r_V|^2|VV\rangle\langle VV|$ reduces then to only one nonvanishing matrix element. The state dependent noise originates from the fact that interference occurs only with a probability according to the quality parameter $\mathcal{Q}'$ and is incoherent otherwise, which finally yields

$$\begin{aligned}\overline{PG}_{\text{mod}}\rho\overline{PG}^{\dagger}_{\text{mod}} &= \mathcal{Q}'(M_{tt} + M_{rr})\rho(M_{tt} + M_{rr})^{\dagger} \\ &\quad + (1 - \mathcal{Q}')(M_{tt}\rho M_{tt}^{\dagger} + M_{rr}\rho M_{rr}^{\dagger}).\end{aligned} \tag{3}$$

From this *ansatz* we obtain a model QPT-matrix χ_{mod} by minimizing the sum of the absolute squared values of all the matrix elements of $\chi_{\text{mod}} - \chi_{\text{exp}}$ numerically [see Fig. 2(c)]. The obtained quality value $\mathcal{Q}' = 0.904$ is in very good agreement with $\mathcal{Q}$ obtained from the fit to the HOM-dip.[c] This indicates that it is indeed mainly imperfect overlap at the beam splitter which causes the state dependent

[c]Other parameters are determined as: $t_V^2/r_V^2 = 2.035$, $a_V t_V = 1.0 a_H t_H$, $b_V t_V = 1.16 b_H t_H$.

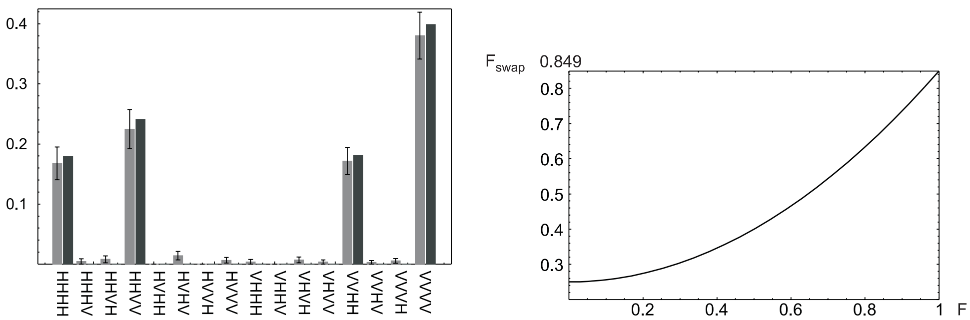
noise. Now we can use the model to predict how well the implemented gate works in real applications for the creation of four photon entanglement.

The C-phase gate has the capability to entangle qubits. In particular it is the operation which applies in the generation of graph states, where qubits are entangled via next-neighbor interaction. Prominent examples of graph states are the so-called cluster states. The cluster state for four photons can be written as:

$$|\mathcal{C}_4\rangle = \frac{1}{2}(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle - |VVVV\rangle). \tag{4}$$

As can be easily seen this state is obtained from the product of two Bell states $|\phi^+\rangle = 1/\sqrt{2}(|HH\rangle + |VV\rangle)$ and the application of our gate on qubits two and three. We have accomplished the corresponding experiment (its detailed description can be found in a recent puplication[8]) and observed the cluster state with a fidelity of $F_{\mathcal{C}_4} = 0.741 \pm 0.013$. Figure 3(a) shows a comparison between the four photon coincidence detection probabilities measured during the experiment and the probabilities obtained from a simulation using χ_{mod}. Taking into account statistical errors caused by finite measurement times, the simulation matches well the experimental results in the high, for the cluster state characteristic, contributions. In particular the consequences of the state dependent noise, leading to an enhancement of the $|VVVV\rangle$ term, can be nicely seen. The small experimental detection probabilities for some of the terms which are not reproduced by the model, result from the fact that the initial Bell states are not prepared with 100% fidelity in the experiment which, in contrast, is assumed for the simulation.

Contrary to the previous considerations, the C-phase gate can also be used for a dis-entangling operation. That enables a complete Bell state analysis (BSA) where the gate transforms between four entangled Bell states and four product states of a particular basis. The BSA is a key element in several quantum communication protocols, like e.g. quantum teleportation[9] or entanglement swapping.[10] In entanglement swapping one starts again from two Bell states and subjects now



(a) (b)

Fig. 3. (a) Measured (grey) and with χ_{mod} simulated (black) four photon detection probabilities for the cluster state $|\mathcal{C}_4\rangle$, (b) Calculated fidelity F_{swap} of the swapped states depending on the fidelity F of the two initial Bell pairs, using χ_{mod} for the BSA.

one photon of each entangled pair to a BSA. As a consequence, the remaining two photons which have never interacted before will be left in one of four Bell states, depending on the outcome of the BSA. Based on our model we can estimate the fidelity of these swapped states in such an experiment. A calculation shows that the quality of the swapped states critically depends on the quality of the initial Bell states. We take that into account, assuming potentially arising white noise in each of the initial Bell states:

$$|\Phi_{\text{in}}\rangle = F\,(|\phi^{+}\rangle\langle\phi^{+}|) + (1-F)\frac{\mathbb{1}^{\otimes 2}}{4}. \tag{5}$$

Figure 3(b) shows the fidelity F_{swap} of a swapped sate to the corresponding pure Bell state depending on the fidelity F of each initial state $|\Phi_{\text{in}}\rangle$. The maximum of F_{swap} achievable with our gate for $F = 1$ is 84.9%. Swapped states with a fidelity larger than $(2+3\sqrt{2})/8 \approx 0.78$ can violate Bell-type inequalities,[11] what requires F to be greater than 94.0%. From an experimental point of view this is a tight constraint but possible to achieve.[12]

To summarize, we have described a recently introduced simple linear optics C-phase gate. We showed that the performed quantum process tomography of our gate in combination with a fit model based on experimental parameters of the setup allows simulations, which reproduce well real experimental data. Therefore it offers a reliable tool for estimating the gate performance in multi-photon experiments what is important for planning of future experiments.

References

1. N. Kiesel, Ch. Schmid, U. Weber, R. Ursin and H. Weinfurter, *Phys. Rev. Lett.* **95** (2005) 210505.
2. I. L. Chuang and M. A. Nielsen, *J. Mod. Opt.* **44** (1997) 2455.
3. J. F. Poyatos, J. I. Cirac and P. Zoller, *Phys. Rev. Lett.* **78** (1997) 390.
4. M. W. Mitchell, C. W. Ellenor, S. Schneider and A. M. Steinberg, *Phys. Rev. Lett.* **91** (2003) 120402.
5. C. K. Hong, Z. Y. Ou and L. Mandel, *Phys. Rev. Lett.* **59** (1987) 2044.
6. R. A. Campos, B. E. A. Saleh and M. C. Teich, *Phys. Rev. A* **42** (1990) 4127.
7. D. F. V. James, P. G. Kwiat, W. J. Munro and A. G. White, *Phys. Rev. A* **64** (2001) 052312.
8. N. Kiesel, C. Schmid, U. Weber, G. Toth, O. Gühne, R. Ursin and H. Weinfurter, *Phys. Rev. Lett.* **95** (2005) 210502.
9. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, *Phys. Rev. Lett* **70** (1993) 1895.
10. M. Zukowski, A. Zeilinger, M. A. Horne and A. K. Ekert, *Phys. Rev. Lett* **71** (1993) 4287.
11. C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin and W. K. Wootters, *Phys. Rev. Lett.* **76** (1996) 722.
12. C. Schmid, N. Kiesel, U. Weber, R. Ursin, A. Zeilinger and H. Weinfurter, Teleportation and entanglement swapping with a complete Bell state analysis, to be published.