Entanglement Persistency of Multiphoton Entangled States

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We experimentally demonstrate the entanglement persistency when losing photons in three- and four-photon polarization-entangled states. The entanglement properties of the mixed states obtained from multiphoton spontaneous parametric down-conversion are studied via witness and positive partial transpose approaches. Together with a quantification of the bipartite entanglement such analysis enables intuitive understanding of novel multiparty quantum communication protocols.

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Entanglement is the key resource of almost all applications of quantum information processing [1]. It is understood to be a very fragile resource which is easily destroyed, particularly when losing particles forming the entangled state. However, this is not necessarily true for the whole variety of multiparticle states. As observed in the ongoing effort to classify multiparty entanglement, there exist multiparticle states which still exhibit a significant amount of entanglement even after the loss of some of the particles. Since particularly bipartite entanglement is well understood (for a review see Ref. [2]), the classification according to the persistent entanglement contained in a state, enables an alternative route for the characterization of multipartite entanglement [3,4].

The analysis of the remaining entanglement opens a new point of view on the underlying principles of multiparty quantum communication schemes. There, the loss of particles corresponds to situations, where the information about particles is not accessible, or, for example, in multiparty quantum cryptography protocols, to the case where some of the parties are not willing to cooperate. The persistency of the entanglement is thus crucial for the implementation and stability of novel protocols.

In this Letter, we experimentally demonstrate the persistency of the entanglement of multipartite entangled states. In particular, we investigate the entanglement properties of the three-photon W state [5] and of the four-photon state $|\Psi^{(4)}\rangle$ [6]. We use three- and two-particle entanglement witnesses and the Peres-Horodecki criterion to confirm the entanglement and use various measures, like the entanglement of formation and the logarithmic negativity to quantify the entanglement of the remaining bipartite states. This, in turn, allows an intuitive analysis of a possible utilization of multipartite entanglement for quantum communication protocols.

The states investigated in this work, the so-called three-photon W state [4]

$$|\mathrm{W}\rangle_{abc} = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle)_{abc} \quad (1)$$

and the four-photon state $|\Psi^{(4)}\rangle$ [6]

$$|\Psi^{(4)}\rangle_{abcd} = \frac{1}{2\sqrt{3}}(2|HHVV\rangle + 2|VVHH\rangle - |HVHV\rangle - |HVVH\rangle - |VHHV\rangle - |VHVH\rangle)_{abcd}, \quad (2)$$

have been observed recently in multiphoton parametric down-conversion experiments [7,8]. Both states are inequivalent to GHZ-type states under stochastic local operations and classical communication [4], and thus allow to perform different quantum communication tasks than the GHZ states. For example, the state $|\Psi^{(4)}\rangle$ is the key resource for tasks like decoherence free quantum communication [9] and quantum telecloning [10].

For the analysis of the three-photon state $|W\rangle$ we note that, due to its symmetry, all the reduced density matrices of two photons are identical; i.e., $\rho_{ab}^{W} = \rho_{ac}^{W} = \rho_{bc}^{W}$. Starting with the density matrix of the pure state $\rho_{abc}^{W} = |W\rangle_{abc}\langle W|$ and tracing over qubit c, one obtains the following two-photon reduced state:

$$\rho_{ab}^{W} = \mathrm{Tr}_c[\rho_{abc}^{W}] = \tfrac{2}{3}|\Psi^+\rangle_{ab}\langle\Psi^+| + \tfrac{1}{3}|HH\rangle_{ab}\langle HH|. \quad (3)$$

with the Bell state $|\Psi^+\rangle = (|HV\rangle + |VH\rangle)/\sqrt{2}$.

The situation is, of course, richer for the four-photon state $|\Psi^{(4)}\rangle$. Firstly, we consider the case where only one photon is lost. The three-photon reduced density matrices are given by $\rho_{abc} = \mathrm{Tr}_d[\rho_{abcd}^{\Psi^{(4)}}] = \rho_{abd} = \mathrm{Tr}_c[\rho_{abcd}^{\Psi^{(4)}}]$ and $\rho_{bcd} = \mathrm{Tr}_c[\rho_{abcd}^{\Psi^{(4)}}] = \rho_{acd} = \mathrm{Tr}_b[\rho_{abcd}^{\Psi^{(4)}}]$, with $\rho_{abcd}^{\Psi^{(4)}} = |\Psi^{(4)}\rangle_{abcd}\langle\Psi^{(4)}|$. These mixed three-qubit states have the same generic structure and thus share the same entanglement characteristics, but due to the form of the state $|\Psi^{(4)}\rangle$ they can be attributed to two different groups. We obtain

$$\rho_{abc} = \rho_{abd} = \frac{1}{2}[|A\rangle\langle A| + |B\rangle\langle B|] \quad (4)$$

where $|A\rangle = (2|HHV\rangle - |HVH\rangle - |VHH\rangle)/\sqrt{6}$ and $|B\rangle = (2|VVH\rangle - |HVV\rangle - |VHV\rangle)/\sqrt{6}$. The other two states can be deduced from these states by permuting indices, i.e., $\rho_{acd} = \rho_{bcd} = \rho_{cab}$ (see Ref. [11]).

Secondly, for the case when two photons are lost, the reduced density matrices fall into two different classes, which significantly differ in their entanglement properties. In the first class, the two photons are lost in modes $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, or $\{b, d\}$, i.e., always one photon from each of the two output channels of the parametric down-conversion [8], and in the second class, the two photons are lost in modes $\{a, b\}$ or $\{c, d\}$. Representatives of the two classes are given by

$$\begin{aligned}\rho_{ab}^{\Psi^{(4)}} &= \mathrm{Tr}_{cd}[\rho_{abcd}^{\Psi^{(4)}}] \\ &= \tfrac{1}{3}|\Psi^{+}\rangle_{ab}\langle\Psi^{+}| + \tfrac{1}{3}(|HH\rangle_{ab}\langle HH| + |VV\rangle_{ab}\langle VV|),\end{aligned} \quad (5)$$

$$\rho_{ac}^{\Psi^{(4)}} = \mathrm{Tr}_{bd}[\rho_{abcd}^{\Psi^{(4)}}] = \frac{2}{3}|\Psi^{-}\rangle_{ac}\langle\Psi^{-}| + \frac{1}{3}\frac{\mathbb{1}_{ac}^{\otimes 2}}{4}, \quad (6)$$

where $|\Psi^{\pm}\rangle$ are Bell states and $\mathbb{1}_{ac}^{\otimes 2}$ is the $4 \otimes 4$ identity matrix. The state $\rho_{ac}^{\Psi^{(4)}}$ is a so-called Werner state, i.e., a mixture of a maximally entangled state, $|\Psi^{-}\rangle$, with the depolarized state, $\mathbb{1}^{\otimes 2}/4$.

For the analysis of the entanglement of these states various tools have been developed. So far, we have state independent criteria and measures only for bipartite systems. We thus use the recently developed entanglement witness to test the entanglement of the three-photon mixed state $\rho_{abc}^{\Psi^{(4)}}$. According to this method a quantum state with the density matrix ρ is entangled iff there exists a Hermitian operator $\mathcal{W}$ such that $\mathrm{Tr}[\mathcal{W}\rho] < 0$, whereas for all separable states σ, $\mathrm{Tr}[\mathcal{W}\sigma] \geq 0$ holds [12–14]. A witness for detecting the tripartite entanglement in the state $\rho_{abc}^{\Psi^{(4)}}$ is $\mathcal{W}_{abc}^{\Psi^{(4)}} = 5/6 \times \mathbb{1}^{\otimes 3} - 2\rho_{abc}^{\Psi^{(4)}}$ [15]. This operator can be decomposed [14], starting from the local decomposition of the witness for the pure state $|\Psi^{(4)}\rangle$ [16] by removing all the terms which contain a (traceless) Pauli matrix for the particle which is traced out, leading to

$$\begin{aligned}\mathcal{W}_{abc}^{\Psi^{(4)}} = \frac{1}{12}\Big(&7 \times \mathbb{1}^{\otimes 3} \\ &+ \sum_{i=x,y,z}[-\sigma_i\sigma_i\mathbb{1} + 2(\sigma_i\mathbb{1}\sigma_i + \mathbb{1}\sigma_i\sigma_i)]\Big).\end{aligned} \quad (7)$$

The theoretical minimum of the expectation value is $\mathrm{Tr}[\mathcal{W}_{abc}^{\Psi^{(4)}}\rho_{abc}^{\Psi^{(4)}}] = -1/6$, clearly signaling the genuine tripartite entanglement of the states $\rho_{abc}^{\Psi^{(4)}}$, etc.

For the representative two-photon reduced states ρ_{ab}^{W} and $\rho_{ac}^{\Psi^{(4)}}$ the witness operators consist of only three local polarization measurements:

$$\mathcal{W}(\rho_{ab}^{W}) = \tfrac{1}{4}(\mathbb{1}^{\otimes 2} - \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z), \quad (8)$$

$$\mathcal{W}(\rho_{ac}^{\Psi^{(4)}}) = \tfrac{1}{4}(\mathbb{1}^{\otimes 2} + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z). \quad (9)$$

The expectation values for the respective states are negative, $\mathrm{Tr}[\mathcal{W}\rho_{ab}^{W}] = -1/6$, and $\mathrm{Tr}[\mathcal{W}\rho_{ac}^{\Psi^{(4)}}] = -1/4$, respectively, indicating the bipartite entanglement of these states. For states of the class of $\rho_{ab}^{\Psi^{(4)}}$ no witness operator can be found, however, since this state is not entangled. To prove its separability the (state-independent) Peres-Horodecki criterion [12,17] is the tool of choice. It is based on the analysis of the density matrix and thus requires experimentally "expensive" full state tomography, but therefore the analysis of the entanglement requires no prior knowledge.

Applying this criterion to the bipartite states obtained from $|W\rangle$ and from $|\Psi^{(4)}\rangle$ under particle loss, we expect negative eigenvalues $\lambda_{ab}^{W} = (1 - \sqrt{5})/6$ and $\lambda_{ac}^{\Psi^{(4)}} = -1/4$ for the partial transpositions, which shows that these states are entangled. If particles c and d are lost, giving the state $\rho_{ab}^{\Psi^{(4)}}$, we find that all eigenvalues of its partial transposed matrix are positive and therefore this state indeed is not entangled. Figure 1. (bottom) illustrates the bipartite entanglement or separability relationships between the particles of the two multiphoton entangled states.

Having collected the necessary tools for evaluating the persistency of multiphoton entanglement, we can turn to the experimental implementation. For the experimental observation of the multiphoton polarization-entangled states, the four-photon emission in two spatial modes during a single pump pulse of type-II spontaneous parametric down-conversion process (SPDC) is used (Fig. 1, top). We used the uv femtosecond pulses of a frequency-doubled

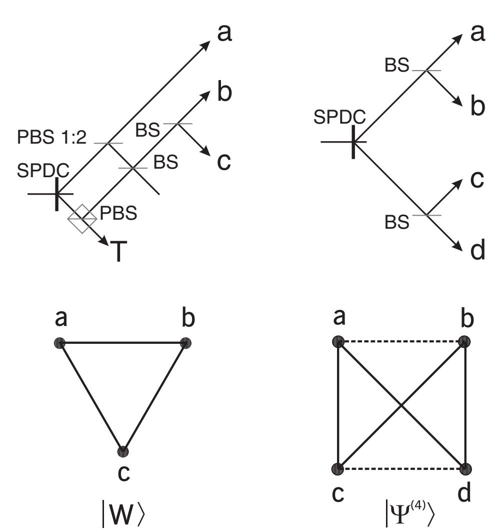

FIG. 1. (top) Schemes of the experimental setups and (bottom) graphs illustrating the bipartite entanglement of (left) $|\mathrm{W}\rangle$ and (right) $|\Psi^{(4)}\rangle$ states. In the graphs the vertices represent qubits, and the solid (dashed) lines represent bipartite entanglement (separability).

mode-locked Ti:sapphire laser to pump a 2 mm thick BBO crystal at a wavelength of 390 nm with an average power of 750 mW. To exactly define the spatial and spectral emission modes the degenerate down-conversion emission was coupled into single mode optical fibers and passed through 3 nm width narrowband interference filters, respectively. To observe the three-photon W state, the four photons are distributed in four modes where the detection of the fourth photon is used as trigger (T). To obtain the coherent superposition in the W state [Eq. (1)] and the correct ratio between H and V polarized photons a polarization dependent beam splitter (PBS 1:2) and a two-photon interference at a BS are employed (for detail see Ref. [7]). In the experimental setup for the observation of the state $|\Psi^{(4)}\rangle$, the four photons are split by two polarization-independent (50:50) beam splitters in each of the two output modes of the source of entangled photons. Entanglement originates in a multiphoton interference in the source causing the emission of two equally polarized photons into one mode more likely compared with orthogonal polarizations [8].

The observation of the states is conditioned on the detection of one photon in each of the four output modes. The polarization measurements are performed in each (except the trigger) mode by a combination of quarter- and half-wave plates, and a polarizing beam splitter. The photons are detected by Si avalanche photodiodes, and the coincidences are registered with an eight channel multi-coincidence logic. We have obtained a rate of 30 per minute fourfold coincidences for the observation of the state $|\Psi^{(4)}\rangle$ and, due to the low efficiency of the linear logic gate producing the W state, only about 2 per minute. For the analysis of the entanglement persistency we register four-photon emissions with a detection event in each of the four modes, and sum over results from observers not contributing to the state under investigation.

Table I presents the experimental results for the evaluation of the three- and two-photon entanglement witnesses. The values obtained by 3 sets of local polarization measurements each (given by $\sigma_i \otimes \sigma_i$, for $i = x, y, z$) clearly indicate the persistency of the entanglement after losing one or even two photons.

For further analysis we have evaluated the two-photon density matrices ρ_{ab}^{W} and $\rho_{ac}^{\Psi^{(4)}}$, by making 16 polarization measurements in different bases of the linear and circular polarizations (H/H, H/V, V/H, V/V, $+45/H$, $+45/V$, $H/+45$, $V/+45$, $+45/+45$, R/H, R/V, $+45/R$, $R/+45$, H/L, V/L, R/L). The results of the measurement allow us to tomographically reconstruct the density matrix of the reduced two-photon states [18]. For example, we have obtained raw data for (H/H, H/V, V/H, V/V) of (685, 2121, 2383, 434) counts in 3 h measurement time. Figure 2 shows the real parts of the elements of the density matrices ρ_{ab}^{W} (a) and $\rho_{ac}^{\Psi^{(4)}}$ (b) in the H/V basis. The imaginary components are on the order of the noise in the real parts and therefore are neglected. The observed reduced two-photon states $\rho^{\rm exp}$ are compared with the

TABLE I. Multiphoton entanglement witnesses.

States	Theory	Experiment
$\rho_{bcd}^{\Psi^{(4)}}$	$-1/6$	-0.116 ± 0.017
$\rho_{acd}^{\Psi^{(4)}}$	$-1/6$	-0.114 ± 0.017
$\rho_{abd}^{\Psi^{(4)}}$	$-1/6$	-0.119 ± 0.017
$\rho_{abc}^{\Psi^{(4)}}$	$-1/6$	-0.112 ± 0.017
$\rho_{ac}^{\Psi^{(4)}}$	$-1/4$	-0.218 ± 0.007
$\rho_{ad}^{\Psi^{(4)}}$	$-1/4$	-0.220 ± 0.007
$\rho_{bc}^{\Psi^{(4)}}$	$-1/4$	-0.211 ± 0.007
$\rho_{bd}^{\Psi^{(4)}}$	$-1/4$	-0.216 ± 0.007
ρ_{ab}^{W}	$-1/6$	-0.065 ± 0.021
ρ_{ac}^{W}	$-1/6$	-0.047 ± 0.025
ρ_{bc}^{W}	$-1/6$	-0.098 ± 0.022

theoretically expected ones $\rho^{\rm th}$ using the fidelity $F = {\rm Tr}[(\sqrt{\rho^{\rm th}}\rho^{\rm exp}\sqrt{\rho^{\rm th}})^{1/2}]^2$. It corresponds to the overlap between the theoretical and experimentally observed states. We obtain $F_{ab}^{W} = 0.876 \pm 0.156$ and $F_{ac}^{\Psi^{(4)}} = 0.978 \pm 0.082$. For the partial transpose of the density matrices ρ_{ab}^{W} and $\rho_{ac}^{\Psi^{(4)}}$, we find negative eigenvalues $\lambda_{ab}^{W} = -0.076 \pm 0.031$ and $\lambda_{ac}^{\Psi^{(4)}} = -0.252 \pm 0.024$, which clearly proves that the two observed states are entangled. All the data presented in this work have been corrected to an average detection efficiency using separately calibrated efficiencies of the various single photon detectors. Errors given are deduced from Poissonian counting statistics of the raw detection events.

For the quantification of the persistent entanglement [19] we list the observed entanglement of formation

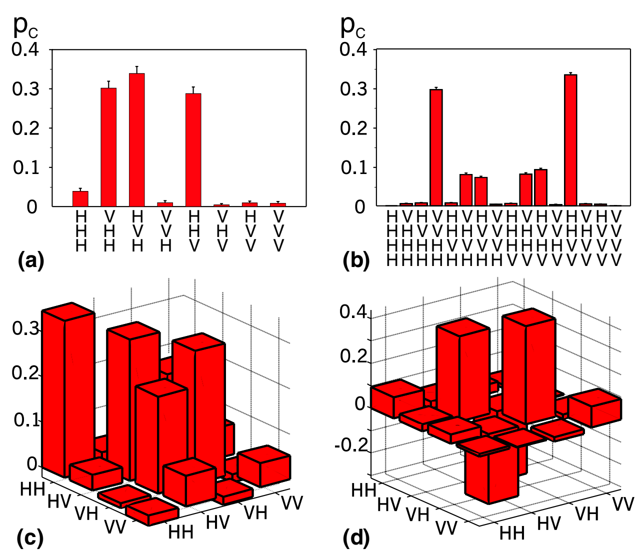

FIG. 2 (color online). Coincidence detection probabilities for the three- and four-qubit states (a) $|W\rangle$ and (b) $|\Psi^{(4)}\rangle$. Real parts of the experimentally determined *two* photon density matrices of the states (c) ρ_{ab}^{W} and (d) $\rho_{ac}^{\Psi^{(4)}}$ in the $\{H, V\}$ polarization basis.

TABLE II. Entanglement of ρ_{ab}^{W} and $\rho_{ac}^{\Psi^{(4)}}$.

states	$E_F(\rho)$	$E_{\mathcal{N}}(\rho)$
ρ_{ab}^{W}	$0.090 \pm 0.057(0.550)$	$0.204 \pm 0.078(0.498)$
$\rho_{ac}^{\Psi^{(4)}}$	$0.362 \pm 0.036(0.355)$	$0.589 \pm 0.046(0.585)$

$E_F(\rho)$ [20] and the logarithmic negativity $E_{\mathcal{N}}(\rho)$ [21] of the reduced two-photon states in Table II. The values in parentheses are theoretical predictions. In theory the W state has very high two-photon entanglement, much higher than the state $|\Psi^{(4)}\rangle$. Yet, the lower coherence due to imperfect overlap at the beam splitter results in a reduction of the persistent entanglement.

Let us now apply the findings to gain better insight into quantum communication protocols. The entanglement of the tri- and bipartite states allows to use these mixed states for quantum cryptography [22]. Thus, contrary to the third man protocols based on GHZ states [23], communication with distributed W states and the states $|\Psi^{(4)}\rangle$ does not require cooperation of all partners. Furthermore, when using these states for quantum secret sharing, the fact that the states have significant bipartite entanglement signals the possibility of using the resulting correlations to obtain partial knowledge about the secret. That means subgroups can already acquire the full secret, which might be of advantage for certain applications.

Using the analysis presented here, one can view quantum telecloning from a new point of view: for a given entanglement fidelity $F(\rho) = \max_{\Phi}\langle\Phi|\rho|\Phi\rangle$ (often also called maximal singlet fraction) one can find a teleportation protocol which achieves an average fidelity of $F_{\rm tel}(\rho) = [2F(\rho) + 1]/3$ [24]. If we apply such a protocol with the reduced two-photon states (6), teleportation fidelities of $F_{\rm tel}(\rho_{ab}^{W}) = 7/9$, $F_{\rm tel}(\rho_{ac}^{\Psi^{(4)}}) = 5/6$, and $F_{\rm tel}(\rho_{ab}^{\Psi^{(4)}}) = 5/9$ can be expected. The last is less than $2/3$ and does not reproduce quantum features, which is consistent with the fact that $\rho_{ab}^{\Psi^{(4)}}$ is not entangled. Note that both the states $\rho_{ac}^{\Psi^{(4)}}$ and $\rho_{ad}^{\Psi^{(4)}}$ simultaneously exhibit the *same* entanglement fidelity. Thus, performing a Bell-state measurement between the photon in arm a and the additional photon carrying the unknown quantum state, we can *simultaneously* teleport this state to *both* of the photons in arms c and d with the same $F_{\rm tel} = 5/6$. Obviously this achieves cloning of the initial state on two remote qubits, characteristic for quantum telecloning [10].

In summary, we have demonstrated experimentally the entanglement persistency of the W state under loss of one photon, and of the state $|\Psi^{(4)}\rangle$ under loss of one or even two photons. Contrary to GHZ states, even after the loss of particles the residual states exhibit significant entanglement. This property makes them useful for novel multiparty quantum communication schemes, for example, for partial knowledge quantum secret sharing or for quantum telecloning.

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