Experimental Realization of a Three-Qubit Entangled *W* State

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We report on the experimental observation of the three-photon polarization-entangled *W* state using spontaneous parametric down-conversion. This state is inequivalent to the Greenberger-Horne-Zeilinger state under stochastic local operations and classical communications and thus is the representative of the second class of genuine tripartite entanglement. We study the characteristic features of entanglement and demonstrate the high degree of two-photon entanglement in the *W* state.

Entanglement plays a central role in the field of quantum information, leading to ongoing efforts for its quantitative and qualitative characterizations. While entanglement of bipartite systems is well understood [1], the characterization of entanglement for multipartite systems is still under intense research. Recently, the equivalence under stochastic local operations and classical communications (SLOCC) was introduced in order to classify multiparty entangled states. This classification is particularly relevant for evaluating their use for multiparty quantum communication. For tripartite systems there are only two different classes of genuine tripartite entanglement, the Greenberger-Horne-Zeilinger (GHZ) class and the *W* class [2,3].

The first class, the GHZ class, is represented by the state $|GHZ\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle)$ [4]. This state is usually referred to as "maximally entangled" in several senses, e.g., it violates Bell inequalities maximally. It is also maximally fragile, i.e., if one or more particles are lost or projected onto the computational basis $\{0, 1\}$, then all the entanglement is destroyed. Experimentally, GHZ states of three photons [5] and three Rydberg atoms [6] were observed.

The representative of the second class is the *W* state [2,7],

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \quad (1)$$

This state, on one hand, shows perfect correlations and violates a three-particle Mermin inequality, but the violation is weaker than for the GHZ state; in this sense, it is less entangled. On the other hand, for the *W* state, two-partite entanglement can be observed after a measurement on one of the particles, contrary to the GHZ state. From this point of view the *W* state is more entangled.

In this Letter we present the experimental observation of the three-photon polarization-entangled *W* state. We demonstrate that polarization measurements on three photons show the characteristic threefold quantum correlations violating a three-particle Mermin inequality. The entanglement between the two photons of the *W* state is analyzed after projection of the third particle onto the computational basis.

Spontaneous parametric down-conversion (SPDC) has been used to create polarization-entangled multiphoton states [5,8]. From the second order emission process of type-II SPDC [9] one obtains four photons emitted into two spatial modes a_0 and b_0. We distribute these photons into four modes a, b, c, and t (Fig. 1). Conditioned on the detection of one photon in each of these, we observe the three-photon *W* state in modes a, b, and c

$$|W\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle_{abc} + |HVH\rangle_{abc} + |VHH\rangle_{abc}). \quad (2)$$

In this notation, $|HHV\rangle_{abc}$ describes the state of one photon in mode a and one in mode b with horizontal polarization, and one in mode c with vertical polarization. In this setup the state forms as follows: if a vertically polarized photon in mode b_0 is reflected at the polarizing beam splitter (PBS) and triggers the detector in mode t, then two of the other three photons from this emission process have horizontal polarization with the third being vertically polarized, as required for the three photons of the *W* state. Also, for each of the three output modes carrying the *W* state, the probability of observing vertical
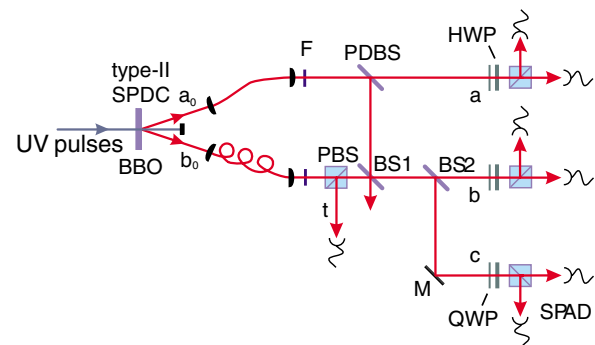

FIG. 1 (color online). Experimental setup for the demonstration of the three-photon polarization-entangled *W* state. For details, see text.

polarization has to be $1/3$. To ensure this probability in case one photon is detected in mode a, a polarization-dependent beam splitter (PDBS) is introduced in mode a_0 with transmissions $T_H = 2T_V$. In order to select the right terms with the correct phases for the W state (2), the photon reflected off the PDBS is superimposed with the horizontally polarized photon in mode b_0 at a 50/50 beam splitter (BS1). Since equally polarized photons exhibit the characteristic bosonic bunching, we cannot use the outputs of BS1 as the final modes, but have to split one of the output modes at another 50/50 beam splitter BS2 into the modes b and c. Thereby we are able to detect a W state with a probability of $1/36$ from each second order SPDC process. We note that there are several other proposals for the generation of W states, but to our knowledge, none has been realized yet [10].

In our experiment, UV pulses with a central wavelength of 390 nm and an average power of 700 mW from a frequency-doubled mode-locked Ti:sapphire laser (pulse length 130 fs) are used to pump a 2 mm thick BBO (β-barium borate) crystal. The degenerate down-conversion emission along the two characteristic type-II crossing directions was coupled into single mode fibers to exactly define the spatial modes, and then filtered with narrow-band interference filters F ($\Delta\lambda = 3$ nm). To align the spatial and temporal overlap of the wave packets at the inputs of BS1 we used Hong-Ou-Mandel interference of photon pairs created in the first order process of SPDC [11]. At the point of maximal two-photon interference, also the required coherent superposition of the three terms forming the three-photon W state is assured. The setup was stable over several hours with typically 125 triggered threefold coincidences per hour. The coincidence count rates had to be corrected for different relative efficiencies of the six detectors in modes a, b, and c. The errors given are deduced from propagated Poissonian counting statistics of the raw detection events and detection efficiencies.

Figure 2(a) shows the eight possible threefold coincidence probabilities for all three polarization analyzers oriented along H/V directions. The coincidence probabilities are defined as $p_{HHH} = c_{HHH}/\sum_{i,j,k=\{H,V\}} c_{ijk}$ etc., where, e.g., c_{HHH} is the number of recorded HHH events. We clearly observe the three-term structure HHV, HVH, and VHH corresponding to a W state (2). The three terms are equally weighted within the experimental errors. However, the same threefold coincidence probabilities could also be obtained by an incoherent mixture $\rho_M = 1/3(|HHV\rangle\langle HHV| + |HVH\rangle\langle HVH| + |VHH\rangle\langle VHH|)$ or by an equally weighted mixture of biseparable states $\rho_B = \frac{1}{3}\rho_a \otimes \rho_{bc} + \frac{1}{3}\rho_b \otimes \rho_{ac} + \frac{1}{3}\rho_c \otimes \rho_{ab}$, with $\rho_a = |H\rangle\langle H|$ and ρ_{bc} a Bell state between modes b and c, etc. We have thus measured the threefold detection probabilities for analysis along left (L) and right (R) circular polarizations [Fig. 2(b)]. For these analyzer settings, ρ_M should exhibit equal probabilities for all the eight threefold detection events, whereas for the state ρ_B the
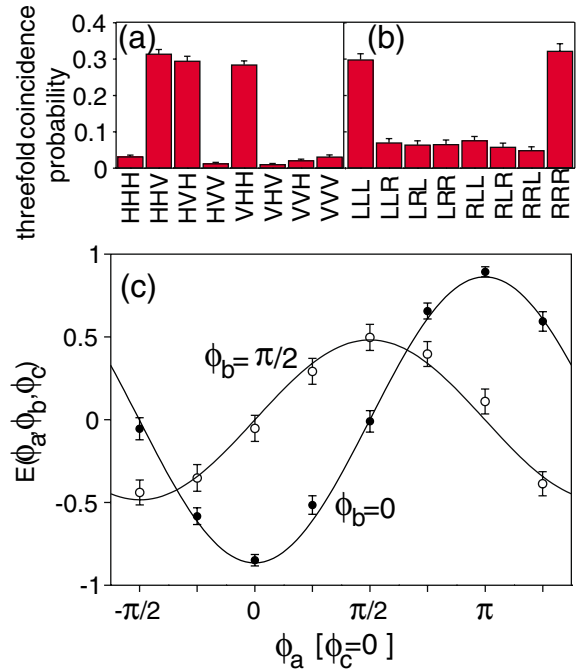

FIG. 2 (color online). Threefold coincidence probabilities measured (a) in the H/V basis showing the three term structure of a W state, and (b) in the L/R basis verifying the superposition of the three terms. (c) Three photon polarization correlation with fixed settings $\phi_b = \phi_c = 0$ (filled circles) and $\phi_b = \pi/2$, $\phi_c = 0$ (open circles) showing a visibility $\mathcal{V}$ of 0.864 ± 0.019 and 0.481 ± 0.029, respectively.

events LLL and RRR should occur with probability $1/4$ and the other events with equal probability $1/12$. Yet, for the W state the probabilities for the LLL and RRR events are $3/8$. Experimentally, we obtain $p_{LLL} = 0.297 \pm 0.017$ and $p_{RRR} = 0.321 \pm 0.021$, respectively, with equal probabilities for the remaining events. These results thus exclude the possible observation of ρ_M or ρ_B.

To experimentally characterize the entanglement of the observed state, correlations between polarization measurement results in the three modes a, b, and c were analyzed. We measured linear polarization given by the eigenstates $|k_j, \phi_j\rangle = 1/\sqrt{2}(|R\rangle + k_j e^{i\phi_j}|L\rangle)$ of the polarization operators $\hat{\sigma}_j = \sum_{k_j} k_j |k_j, \phi_j\rangle\langle k_j, \phi_j|$, where $k_j = \pm 1$ denotes the local results in mode $j = a, b, c$. The correlation function for three photons is defined as the expectation value

$$E(\phi_a, \phi_b, \phi_c) = \langle \hat{\sigma}_a(\phi_a)\hat{\sigma}_b(\phi_b)\hat{\sigma}_c(\phi_c)\rangle$$
$$= \sum_{k_a, k_b, k_c = \pm 1} k_a k_b k_c \, p_{k_a k_b k_c}(\phi_a, \phi_b, \phi_c), \quad (3)$$

where $p_{k_a k_b k_c}(\phi_a, \phi_b, \phi_c)$ is the probability for a threefold coincidence with the results k_a, k_b, and k_c for the specific setting of phases. For the W state one analytically obtains

$$E(\phi_a, \phi_b, \phi_c) = -\frac{2}{3}\cos(\phi_a + \phi_b + \phi_c)$$
$$- \frac{1}{3}\cos(\phi_a)\cos(\phi_b)\cos(\phi_c). \quad (4)$$

TABLE I. Mermin inequality.

(ϕ_a, ϕ_b, ϕ_c)	$(\pi/2, \pi/2, 0)$	$(\pi/2, 0, \pi/2)$	$(0, \pi/2, \pi/2)$	$(0,0,0)$
$E(\phi_a, \phi_b, \phi_c)$	0.529	0.569	0.519	-0.851
ΔE	0.024	0.046	0.032	0.014
		$S_{\exp}^{M} = 2.468 \pm 0.063$		

For settings $\phi_b = \phi_c = 0$, the correlation function simplifies to $E(\phi_a, 0, 0) = -\cos(\phi_a)$ and exhibits perfect correlations. A sinusoidal fit to the experimentally obtained correlations [Fig. 2(c)] gives a visibility $\mathcal{V} = 0.864 \pm 0.019$ [12], compared to $\mathcal{V} = 1$ for a pure W state. Additional evidence for its observation was obtained by measuring the correlation function for the fixed settings $\phi_b = \pi/2$, $\phi_c = 0$ [Fig. 2(c), open circles]. The theoretical value for the visibility of this correlation function is $2/3$. An incoherent mixture ρ_M would not show any correlations, and the mixture of biseparable states ρ_B has a visibility of only $1/3$. A sinusoidal fit to the experimental data gives the visibility $\mathcal{V} = 0.481 \pm 0.029$ [12]. This demonstrates again the coherence of the three terms forming the W state. In contrast to the W state, a GHZ state does not exhibit any correlations for such analyzer configurations $[E_{\mathrm{GHZ}}(\phi_a, \phi_b, 0) = 0]$. However, it shows perfect correlations if $\phi_b = \phi_c = \pi/2$, whereas $|E_W(\phi_a, \pi/2, \pi/2)| < 2/3$ for the W state [not shown in Fig. 2(c)]. These results therefore reveal different types of correlations for this second class of genuine tripartite entanglement.

To further characterize the entanglement of the observed state, we tested an inequality derived by Mermin [13]

$$S^M = E(\phi_a, \phi_b, \phi_c') + E(\phi_a, \phi_b', \phi_c) + E(\phi_a', \phi_b, \phi_c)$$
$$- E(\phi_a', \phi_b', \phi_c')$$
$$\leq 2. \qquad (5)$$

Therein, ϕ_j, ϕ_j' are the two possible analyzer settings of the three observers $j = a, b, c$. One can use the value S^M to investigate the entanglement of the state. For separable states, S^M is bound by 2. The highest violation of this inequality is obtained for the GHZ state ($S_{\mathrm{GHZ}}^M = 4$), which is the reason for the GHZ state usually being referred to as maximally entangled. The maximal value for the W state is $S_W^M = 3.046$ [14] for analyzer settings $\phi_j = 0.943$ rad and $\phi_j' = 2.725$ rad, respectively. This still exceeds the bound for $2\sqrt{2}$ of biseparable states, revealing the genuine tripartite entanglement of the W state. For convenience we have chosen experimental settings $\phi_j = \pi/2$, $\phi_j' = 0$, which results for a pure W state in a value of $S_W^M = 3$. Table I shows the measured values for the correlation functions required to evaluate this inequality. The resulting violation with $S_{\exp}^M = 2.468 \pm 0.063$ is clearly above 2, but does not exceed the bound of $2\sqrt{2}$. This does not mean that there is no tripartite entanglement in the observed state. In fact, the measure-

ments shown in Fig. 2 strongly indicate its presence. Unambiguous evidence for tripartite entanglement can be obtained using witness operators [15].

Another interesting feature of the W state is the robustness of the entanglement. Whereas for the GHZ state the entanglement is fragile and completely lost after a projection measurement in the H/V basis, there is still two-photon entanglement left for the W state. If the polarization of one photon, e.g., in mode c, is measured, and the result is $'H'$ (which happens with probability $2/3$), the remaining two photons are projected into a maximally entangled Bell state, $|\Psi^H\rangle_{ab} = 1/\sqrt{2}(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)$. On the contrary, if the measurement yields $'V'$, they are projected into a product state $|\Psi^V\rangle_{ab} = |H\rangle_a|H\rangle_b$. The W state thus has higher two-photon entanglement than the GHZ state. Figure 3 shows the two-photon correlation functions for the photons in modes a and b, depending on the measurement result of the photon in mode c. As expected, there are strong correlations corresponding to the maximally entangled state $|\Psi^H\rangle_{ab}$ ($\mathcal{V} = 0.859 \pm 0.045$), whereas there are no correlations conditioned on detection of $'V'$ ($\mathcal{V} < 0.05$). The correlations between measurement results of two photons conditioned on the detection of $'H'$ in mode c allowed us to violate a two-particle Clauser-Horne-Shimony-Holt (CHSH) inequality [16]. Experimentally, we found the value $S_{\exp}^{\mathrm{CHSH}} = 2.509 \pm 0.094$, which is well above the bound of 2 for separable states (see Table II). This demonstrates
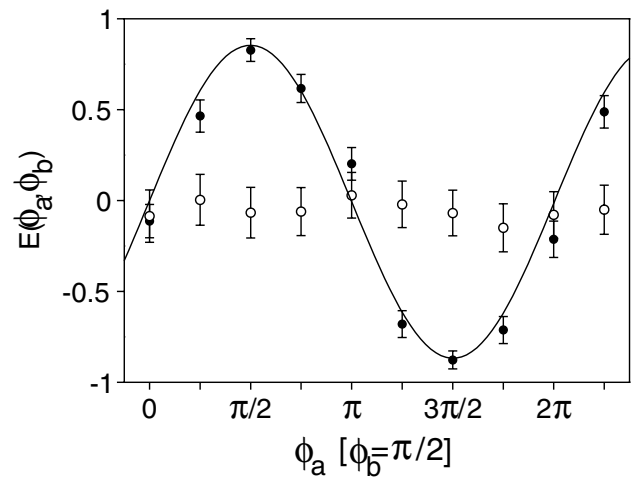

FIG. 3. Correlations between photons in modes a and b, conditioned on the detection of a H-polarized photon (filled circles) or the detection of a V-polarized photon (open circles) in mode c.

TABLE II. Conditional two-photon CHSH inequality.

(ϕ_a, ϕ_b)	$(0, -\pi/4)$	$(\pi/2, -\pi/4)$	$(0, \pi/4)$	$(\pi/2, \pi/4)$
$E(\phi_a, \phi_b)$	-0.693	-0.523	-0.656	0.637
ΔE	0.033	0.048	0.047	0.056
	$S_{\exp}^{\mathrm{CHSH}} = 2.509 \pm 0.094$			

clearly the two-photon entanglement conditioned on the measurement result $'H'$.

An alternative way to analyze bipartite entanglement is to use the Peres-Horodecki criterion [1]. To apply this criterion, we have performed quantum state tomography to determine the density matrices ρ_{ab}^H and ρ_{ab}^V of the photons in modes a and b, conditioned on the measurement results $'H'$ and $'V'$ in mode c. Figure 4 shows the real parts of the experimental density matrices. The imaginary parts are on the order of the noise in the real parts.

The smallest eigenvalues of the partial transpose of the experimental density matrices are $\lambda_H^{\exp} = -0.348 \pm 0.019$ and $\lambda_V^{\exp} = -0.113 \pm 0.062$, respectively, compared to the theoretical values of $\lambda_H = -0.5$ and $\lambda_V = 0$. Applying the Peres-Horodecki criterion, the negative eigenvalue $\lambda_H^{\exp}$ clearly reveals the entanglement conditioned on the detection of $'H'$. Yet, $\lambda_V^{\exp}$ differs from zero by almost 2 standard deviations, suggesting entanglement of the remaining photons, too. Most probably the asymmetry of the setup also causes experimental imperfection not to be modeled as white noise, resulting in different admixtures of entangled states. Those, however, must be so small that they do not show up in the correlation function (Fig. 3).

In this Letter the observation of the three-photon polarization-entangled W state was reported. The analysis performed shows that the observed state is not compatible with incoherent mixtures or specific biseparable states, but that it has genuine three-photon entanglement. The W state is the representative of the second class of tripartite entanglement under SLOCC and has remarkably different entanglement properties, when compared to
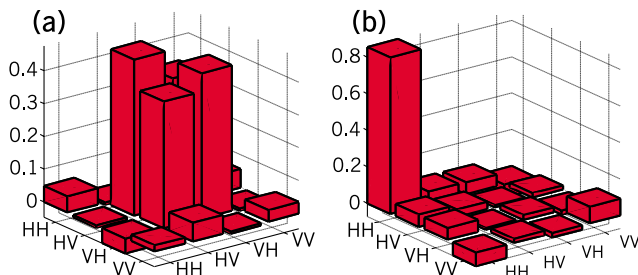
the GHZ state. The W state does not exhibit a set of four perfect correlations to violate the Mermin inequality as strongly as a GHZ state. Nevertheless, it was shown here that correlations are observed in other analysis directions, and that it contains substantial two-photon entanglement when one of the three photons is measured in the computational basis, or even is lost. Besides possible applications of the W state for secret sharing or quantum teleportation [17], the knowledge of its characteristic features is important for better understanding entanglement of many particles in general.

FIG. 4 (color online). Real parts of experimentally determined density matrices of the two photons in modes a and b, conditioned on the detection of a (a) H-polarized photon or (b) V-polarized photon in mode c.

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