Two-Photon Polarization Entanglement in Typ I SPDC Experiment

Bachelor Thesis

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1 Motivation and structure of the study

Motivation Certainly, quantum mechanics is one of the most successful physical theories. When quantum theory was developed during the first decades of the 20th century, it step by step provided understanding of a number of physical phenomena, which could not be explained by any of the so called classical theories. The new theory was able to predict, for example, the atomic line spectrum of hydrogen with astonishing accuracy. Paradoxes like the one occuring in the famous double-slit experiment, could be resolved by application of the new principles[1].

Until now, classical computation has been a fast developing field for several decades. Today, we can perform complicated calculations in a few seconds on handy notebooks. Only few decades ago, a whole house full of computers was needed for the same exercise. Currently, however, computation approaches a new frontier. Since the transistors needed to ever increase the computational power, have become smaller and smaller, they eventually must reach a size, in which quantum effects become important. Although there may be found methods in the future, to handle these effects, at the latest zero will set an lower bound on the size. Therefore it is important to find new paths, along which computation can be further developed. "Smaller and smaller still" will not work forever.

The research on the field of modern optics has lead to a deep insight in optical effects. Today, lasers can be driven stable at very high intensity. Therefore, it is possible to examine the interaction of light with matter at high field strength. These experiments have lead to the discovery of so called nonlinear effects like parametric fluorescence[2].For this phenomenon also, quantum mechanics provides a rather elegant description in the second quantized language[3]

Nonlinear optics and classical computation are very important mosaics in the field of quantum information. Quantum information emerged from the try to find new methods of computation. Having developed for the last fourty years, quantum information has become a rich and interesting realm of modern physics. It contains many fields of interest[1]. One of them deals with the search for efficient bit encoding. In classical computation, one bit, carrying the information "one" or "zero", may for example be represented by "current" and "no current". Therefore, one bit always carries defined information, either "one" or "zero". In quantum information, one approach is to use as bits photons with their two possible orthogonal polarizations. Even more, one tries to encode the information in pairs of photons. It turns out

[1] A general treatment on the fundamentals of quantum information is provided in [4]

that photon pairs can be prepared in highly correlated states, called entangled states, in which they can carry more information per particle, than in any classical case[5]. This motivates the search for efficient sources of entangled photon pairs. The problem is actually twofold. First, one has to search for an appropriate method to produce these pairs. Research has developed many different methods of quite high efficiency, mainly based on parametric fluorescence. Second, one needs experimental tests, to prove the actual existence of the desired correlations between the photons of a pair. Many criteria have been proposed, from which experimental tests can be designed to detect the produced entanglement.

Abstract On the one hand, this work deals with creation and detection of entangled two-photon states. To produce entangled photon pairs, a very efficient source was used, based on a particular kind of parametric fluorescence. It shall be discussed, under what conditions entanglement will be produced. The successful production is to be proven by application of several experimental tests. On the other hand, the notion of decoherence of entangled states will play an important role in course of this work. It shall be illustrated, how the correlations between the photons of a pair can be diminished in a controlled way, to finally destroy entanglement.

Structure The structure of this work can be divided into three parts.
The first part contains a detailed description of the quantum mechanical foundations, on which this study is based. The representation of physical states as vectors in Hilbert space is central in this treatment. A definition of separability, and an identification of entangled states with non-separable states is provided. Furthermore, the famous EPR paradox and the Bell inequalities are discussed. Moreover, a treatment on density operators is included. The first part is concluded by a synopsis of criteria, which were used to detect entanglement of the states prepared in this study.
The second part shall serve as a detailed overview of the experimental concepts on which this sudy is built up. The creation of correlated photon pairs is described, where value is placed on typ I SPDC, since it was central in this work. Next to that, an extensive description of state tomographies is incorporated. Within, besides an explanation of the basic principles of state tomography, the representation of two-photon polarization states is introduced. As a special case of the class of projection measurements, the projection on separable two-photon polarization states is discussed in great detail. These projections form the basis of the tomographies carried out in this study. The second part is completed by a treatment on the walk-off

effects in uniaxial crystals.
A detailed presentation of the experiments performed in course of this work is subject of the third part. To start with, the experimental setup, which was used in this work, is described in great detail. Great importance was attached to the preparation of entangled states in this implementation. Hereafter, the depiction of a correlation measurement on the two-photon state is supplied. Subsequently, an experiment carried out to violate the CHSH inequality is outlined. In both experiments the results and their interpretation are exposed as well. Subsequently, the state tomography performed to reveal the form of the prepared state is presented. The application of entanglement criteria to the prepared state is described. The third part is concluded with the decoherence experiment, which was performed to show the connection between temporal separation of the photon pairs and destruction of entanglement.

2 Foundations of quantum mechanics

2.1 State vector representation of quantum states

Physical states as vectors in Hilbert space In quantum theory, physical states are postulated to correspond to normalizable vectors $|\phi\rangle$ on a Hilbert space $\mathcal{H}$[6]. Note that Hilbert space is a special case of linear vector spaces. The dimensionality of the space equals the number of physically allowed distinct configurations of the described system. In the case of only two possible configurations, one can describe the system in the orthonormal basis $\{|a\rangle, |b\rangle\}$. Since $\mathcal{H}$ is a vector space, also any normalized superposition

$$|\phi\rangle = \alpha\,|a\rangle + \beta\,|b\rangle\,, |\alpha|^2 + |\beta|^2 = 1 \tag{1}$$

is an element of $\mathcal{H}$ and can therefore, according to quantum mechanics, describe a physical state of the system[2]. Note that it is indeed possible to prepare the system in such a superposition state. One important remark must be made on the physical nature of such a state.
Consider a measureable quantity, like the energy E. Assume that the system has energy $E = E_a$ in the state $|a\rangle$, and energy $E = E_b$ in the state $|b\rangle$. Now, the system shall be prepared in a superposition state of the form (1). If one measures the energy of the system, one will obtain $E = E_a$ and $E = E_b$ with probabilities $|\alpha|^2$ and $|\beta|^2$. That means, that although the state of the

[2]In course of this study, all state coefficients are complex numbers and meet the normalization requirement

system is still perfectly well defined, the outcome of measurements is not deterministic!

Tensor product of Hilbert spaces Consider n non-interacting systems S_i, which may be represented by state vectors $|S_i\rangle$ in Hilbert spaces $\mathcal{H}_{S_i}$. To treat the systems S_i as a single compound system S, one must specify a rule how to assign a state vector to S. The notion of tensor products provides such a rule[4]. The following property of the tensor product is fundamental for the subsequent discussion.

Consider two Hilbert spaces $\mathcal{H}_1, \mathcal{H}_2$. Then the tensor product, denoted by $\mathcal{H}_1 \otimes \mathcal{H}_2$, can be constructed. The following property of the tensor product is fundamental for the subsequent discussion.
Let $\mathcal{H}_1, \mathcal{H}_2$ two Hilbert spaces, on which bases $\{|i_1\rangle\}, \{|j_2\rangle\}$ can be established. Then $|i_1\rangle \otimes |j_2\rangle$ is a basis of the tensor product $\mathcal{H}_1 \otimes H_2$.
Therefore, there can be assigned a state vector to S, which is defined on the multiple tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes ... \otimes \mathcal{H}_n$, and takes the following form

$$|S\rangle = |S_1\rangle \otimes |S_2\rangle \otimes ... |S_n\rangle := |S_1, S_2, S_3, ..., S_n\rangle \tag{2}$$

Separability and entanglement Note that it is not in general possible to factorize the state $|S\rangle$ of a given compound system S into the form (2).This motivates the following definition.

Consider a system S, built up of subsystems S_i, in the state $|S\rangle$. $|S\rangle$ is called separable, if it is possible to factorize $|S\rangle$ into a product of subsystem states $|S_i\rangle$. Otherwise it is called non-separable.

As an example of a non-separable state of a bipartite system, consider the so called singlet state[3]

$$|\phi\rangle = \frac{1}{\sqrt{2}} (|a, b\rangle - |b, a\rangle) \tag{3}$$

The states $|a\rangle, |b\rangle$ represent "spin up" or "spin down", respectively. The physical meaning of the state introduced in (3) is the following.

Consider a bipartite system in this state. Assume that a spin measurement of one of the particles, performed in a chosen spatial direction, yields the

[3] More precisely, the spin singlet of a bipartite spin $\frac{1}{2}$ system.

result "up". Then one can predict with certainty the outcome of a spin measurement of the other particle, carried out in the same direction. The other particle will, independent of the chosen direction, always come out "down".

Therefore, the particles exhibit very strong correlations, and Erwin Schrödinger introduced the new notion of entanglement[7] to describe such kind of correlations. The meaning of entanglement can be explained in a prosaic way.

If in a compound system the subsystems may have interacted in the past, the state of the compound system is in general not separable. If there exists or existed interaction between the subsystems, the subsystems cannot be described independently of each other. They are entangled.

2.2 EPR paradox and Bell inequalities

The EPR paradox In 1935, EPR[4] published a paper in which they suspected the completeness of quantum theory. They queried whether quantum theory could provide a full description of physical reality in nature. Their reasoning can be explained, if one sets up from the previous spin experiment. Consider again a measurement of spin in a certain direction. Quantum mechanics tells us only how to compute the probability to measure one of the both particles to be "spin up". It negates the possibility, that this result may be already uniquely predetermined and is only revealed in the measurement. Nevertheless, if we measure the spin of one particle in some direction and get a certain result, we know for sure that, for the other particle the opposite result would have been found. This seemed somewhat paradox to EPR. They were convinced that any complete physical theory must incorporate the principles of reality and locality, which read, applied to the concerned experiment[4]

1. principle of reality. Both particles of the considered bipartite systems have intrinsic values of any measurable physical quantity, independent of measurements.

2. principle of locality. A measurement on one particle can only influence the intrinsic values of the other particle in a causally connected way.

EPR concluded that, to ensure completeness, quantum mechanics must incorporate According to EPR, to ensure completeness of quantum theory,

[4] Albert Einstein, Boris Podolsky and Nathan Rosen, [8]

it must therefore incorporate so called hidden[5] variables, from which these intrinsic values can be computed.

Bell Inequalities Therefore, the search for such LHV[6] theories subsequently became a central subject of research. Nevertheless, there was no apparent method to check experimentally the principles of reality and locality in this context. In 1964, however, John Bell found his famous theorem, in which he proved the existence of inequalities, which have to be satisfied within any LHV theory[9]. A further step was made in 1969, when Clauser, Horne, Shimony and Holt proposed an experiment designed to ask nature for a decision between quantum mechanics and the class of LHV theories[10]. The experiment is based on the so called CHSH inequality. This inequality relates expectation values, which shall be obtained experimentally in a two-photon polarization measurement. A violation of the CHSH inequality would violate human intuition as well, since it would imply that either locality or reality, if not both, must be rejected as fundamental features of nature. A detailed description of this test is reserved for the experimental part of this work.

2.3 Density operator representation of quantum states

The state of a physical system S is fully determined by the knowledge of the density operator ρ_S. To represent systems by their density operator is often useful and in some sense more practical than the representation by state vectors. Even more, there actually exist situations in which it is impossible to assign a state vector to S, which correctly represents the state of the system. This system is then said to be in a mixed state. In such cases, only the concept of density operators provides a full description of S. As already mentioned, the notion of density operators is central in this study. This motivates a detailed treatment on density operators, which reaches from a compilation of the general properties, over the cases of pure and mixed states, to an important discussion on quantum correlations.

General properties of density operators Density operators must fulfill the following conditions.

1. Normalization: $Tr(\rho) = 1$
2. Hermiticity: $\rho_{ij} = (\rho_{ji})^*$

[5]not directly measurable
[6]local hidden variables

3. Non-negative definiteness: $\forall |\Psi\rangle : \langle\Psi|\rho|\Psi\rangle \geq 0$

Pure states A physical system C is in a pure state, if it can be completely described by a state vector $|\Psi_C\rangle$. In this case the density operator of C equals the projector on this state.

$$\rho_C = |\Psi_C\rangle\langle\Psi_C| \tag{4}$$

It follows that for pure states

$$Tr(\rho^2) = Tr(\rho) = 1 \tag{5}$$

An advantage of using the density operator for the description of a pure state $|\Psi_C\rangle$, lies in the fact that it remains the same for states which differ from $|\Psi_C\rangle$ only by a global phase factor $e^{i\phi}$. Global phases are unobservable and have no physical relevance. Expectation values for measurements of some observable A are then given by

$$\hat{A} = Tr(\rho A) \tag{6}$$

Mixed states A system S is said to be in a mixed state, if there exist a convex set of $\{p_i\}$, $p_i \geq 0$, $\sum p_i = 1$, such that

$$\rho_S = \sum p_i \rho_i \tag{7}$$

where the ρ_i are projectors onto distinct, but not necessarily orthogonal states. This expression is interpreted as follows. Assume an infinite number of identical copies of S, which are non-interacting. This infinite number of identical copies constitutes an ensemble in statistical physics. If a measurement of the state of all members of the ensemble is performed, a fraction p_i will appear to be in state ρ_i. Note that the expression for expectation values given in the pure case still holds for mixed states, but is now termed ensemble average since the value is found by twofold averaging. The expectation values in the states ρ_i must be computed and then be averaged with respect to the weights $\{p_i\}$.

Quantum correlations in entangled states The existence of quantum correlations is one feature of entangled states, which may serve as a alternative definition of non-separability. Quantum correlations, which are often refered to as coherences, shall be defined in this work as follows[6].

Quantum correlations or coherences are non-diagonal entries of the state operator ρ, which cannot be removed by basis changes.

To illustrate the meaning of this definition, consider first the linear superposition (1) and construct the density operator of this state. Note that (1) is a pure state. Note further that any comlex number $Z = A + iB$ can be written

$$Z = \tilde{z} \cdot e^{i\phi} \tag{8}$$

where $\tilde{z} := \sqrt{A^2 + B^2}$ and ϕ is an appropriate real phase. Then the density operator ρ is readily computed to be

$$\rho = \tilde{\alpha}^2 \cdot |a\rangle \langle a| + \tilde{\beta}^2 \cdot |b\rangle \langle b| + \tilde{\alpha}\tilde{\beta}e^{i\phi} \cdot |a\rangle \langle b| + \tilde{\alpha}\tilde{\beta}e^{-i\phi} \cdot |b\rangle \langle a| \tag{9}$$

There contribute two kinds of terms to ρ, which have very different meanings. The first two terms on the right side of (8) are called populations[7]. The last two terms display quantum interference effects. It is apparent that this interference occurs only if the state of the system is in some superposition. Note that the interference can be removed in the considered case by a change of basis. One can choose a basis $|c\rangle, |d\rangle$, where $|c\rangle = \alpha |a\rangle + \beta |b\rangle$. The same argument can be carried over to a general product state as follows. There certainly exists a set of basis transformations, one defined on each of the subspaces, such that the considered product state takes diagonal form in the new product basis. Therefore the coherences have no invariant meaning in the case of product states. They can always be removed by some appropriate change of basis. Nevertheless, this is not possible for entangled states. Following the definition of separability, there exists no product basis to describe entangled states. Therefore it is meaningful to speak of quantum correlations in this cases, since they only occur within quantum theory.

2.4 Entanglement criteria

To prove that a system was prepared in an entangled state, several criteria can be used. On the one hand, there exist criteria based on direct analysis of the density operator of the system. To apply such kind of criterion, the density operator of the system must be measured, what is done by use of so called state tomographies. On the other hand, the class of correlations measurements provide a method to detect quantum correlations and thus the non-separability of the state without direct knowledge of the density operator. As to be shown, measures of the quality of the produced state are available as well.

[7] recall the interpretation of the coefficients $\tilde{\alpha}^2$ and $\tilde{\beta}^2$

Correlations measurements As already discussed, entangled states show particularly strong correlations, independent of the basis in which the analysis is carried out. It follows that if measurements are performed in a given basis and correlations are found, the same measurements can be repeated in different bases to check if the correlations stay present. This procedure therefore serves as a first criterion of separability and will be took up again in the experimental part of this work.

Peres-Horodecki criterion Since this study is only concerned with two-photon states, only bipartite systems are considered. In this case the criterion is necessary and sufficient[11]. It is based on positive partial transpose(PPT). Before the criterion is posed, the underlying mathematical notions are reviewed. In a given product basis, every bipartite density operator can be written

$$\rho = \sum_{i,j} \sum_{k,l} \rho_{ij,kl} \left| i \right\rangle \left\langle j \right| \otimes \left| k \right\rangle \left\langle l \right| \tag{10}$$

Partial transposition is defined as the transposition of ρ with respect to one subsystem. For example, ρ^{T_a} is defined

$$\rho^{T_a} = \sum_{i,j} \sum_{k,l} \rho_{ji,kl} \left| i \right\rangle \left\langle j \right| \otimes \left| k \right\rangle \left\langle l \right| \tag{11}$$

PPT then means that the transposition with respect to one subsystem is positive semidefinite. Peres and Horodecki proved the following.

A bipartite state ρ is separable, if and only if ρ is PPT.

This is a very strong criterion,since it provides a complete characterization of bipartite entanglement.

Fidelity To determine the overlap between the measured state, ρ, and a theoretical predicted state, $|\Psi\rangle$, the fidelity is a good criterion. It is defined

$$F = \langle \Psi | \rho | \Psi \rangle \tag{12}$$

Purity The purity p is defined under the assumption, that in the experiment the state

$$\rho = p \left| \Psi \right\rangle \left\langle \Psi \right| + (1 - p)\sigma \tag{13}$$

was prepared, where $|\Psi\rangle$ is the desired state and σ represents some noise.

Measure of mixture The trace of ρ^2 is a measure of mixture. It provides a criterion to determine to what extend the prepared states fails to be pure. However, it tells nothing about entanglement.

3 Basic experimental concepts

3.1 Creation of correlated photon pairs in SPDC

In the future, entangled two-photon polarization states may be used for quantum information. Therefore, it is a topic of current research to develop high efficient sources of correlated photon pairs. One obtains highly correlated pairs of particles in decays. This is because such decays obey a number of conservation laws. It is thus natural, to look for decays in which photon pairs are produced. The most appropriate decay by far is spontaneous parametric down-conversion (SPDC). Provided that certain requirements are met, photons can decay in dielectrics into a pair of secondary photons of lower energy. In the following, these requirements are discussed in some depth.

Nonlinear media Consider a dielectric medium which is exposed to a laser beam. The electric field propagating with the laser beam leads to a polarization of the dielectric[2]. Since SPDC appears to be a nonlinear effect, it can occur only if the second-order contribution to the polarization of the medium is nonzero. This implies a finite second-order suszeptibility of the medium and sufficiently high field strength. Media which possess nonzero second-order suszeptibility, are called nonlinear.

Conservation of energy Energy conservation implies, that the energy of the produced photon pair equals the energy of the disintegrated photon. Therefore,

$$\omega_0 = \omega_1 + \omega_2 \tag{14}$$

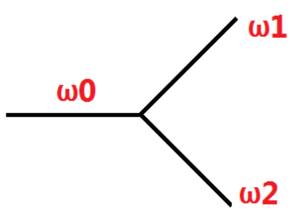


Figure 1: energy conservation

Phase matching SPDC can only be observed if the phases between the interacting fields[8] are matched. Otherwise, down-conversion at any place in the medium will interfere destructively with down-conversion at some other place. That means that there does no SPDC will happen at all. The phase matching condition can be written in the form of the wave vectors

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2 \tag{15}$$

There exist several possibilities to achieve phase matching[3]. In this study, Beta-barium borate, which is a negative uniaxial crystal which exhibits strong birefringence, is used. In that case, phase matching is ensured if the pump photon is extraordinary polarized and the downconversion photons emerge either both ordinary polarized (type I SPDC) or orthogonal to each other (type II SPDC). Typ I phase matching can be obtained over quite broad spectral range and is therefore easier to realize than type II[3]. Therefore, the former possibility was chosen in this study. The implementation used will be described in detail in the experimental part of this work.

3.2 State tomography of two-photon polarization states

Consider a collection of n photons constituting a physical system, which is prepared in some polarization state Φ. If one aims at detecting this state, one certainly needs n polarization analyzers. A rule is required which defines what measurements must be performed. One the one hand, this rule should be efficient in the sense that the number of measurements be as small as possible. On the other hand, it should be simple with respect to the calculations, which are eventually needed to reconstruct the state from the results of the measurements. There exist such rules called state tomographies, based on a complete set of projective measurements, which are applicable to any number of photons. In this study, however, only two-photon states are considered[9]. The state tomographies in this work are carried out according to the method proposed in []. The idea is to reconstruct the density operator from the results of a complete set of 16 projective measurements, which constitute the components (n_ν) of a so called polarization vector of the system. A set of 16 matrices $\left(\hat{M}_\mu\right)$, based on tensor products of the complete set of pauli matrices, is established such that the density operator ρ is given in terms of the matrizes $\left(\hat{M}_\epsilon \cdot n_\epsilon\right)$. To apply this method, one first needs to know

[8]Interactions between pump field and fields of the down-conversion photons

[9]The discussion carries over directly to the description of any number of photons in the system

how to represent such two-photon polarization states quantum-mechanically. Furthermore, information is needed on how to set up any of the projections experimentally by use of optical devices. This subsection shall provide a general treatment on representation of two-photon polarization states. This is followed by a quantum-mechanical description of projections on separable two-photon polarization states. Subsequently, the polarization states to be prepared in the experiments are used as an example to show, how such a projection measurement works.

3.2.1 Two-photon polarization states

Single photon polarization Photon polarization states may be described on a two-dimensional Hilbert space $\mathcal{H}$ by vectors

$$|\Psi\rangle = h\,|H\rangle + v\,|V\rangle \tag{16}$$

where $|H\rangle$ and $|V\rangle$ characterize horizontally and vertically polarized photons, respectively. The orientation of the H-axis in space is arbitrary.

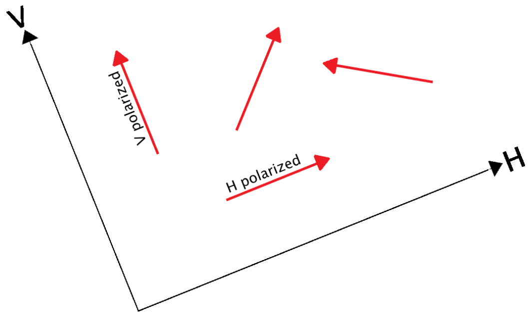


Figure 2: arbitrary choice of polarization basis

It turns out that one can describe the polarization in terms of linear and circular polarized states, dependent on the basis chosen. The most convenient bases, which are also central in this work, are the following.

1. HV linear $\{|H\rangle, |V\rangle\}$
2. $D\bar{D}$ linear $\left\{\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)\right\}$
3. RL circular $\left\{\frac{1}{\sqrt{2}}(|H\rangle - i\,|V\rangle), \frac{1}{\sqrt{2}}(|H\rangle + i\,|V\rangle)\right\}$

Two-photon polarization Since this study is concerned with the creation of photon pairs, it is important to define the representation of two-photon polarization states. Consistent with previous discussion, any two-photon state can be expressed in the tensor product $\mathcal{H} \otimes \mathcal{H}$ in terms of the basis vectors $\{|HH\rangle, |HV\rangle, |VH\rangle, |VV\rangle\}$, where the first and second entries correspond to photon one and two, respectively. The most general state is given by

$$|\Psi\rangle = a\,|HH\rangle + b\,|HV\rangle + c\,|VH\rangle + d\,|VV\rangle\,, \tag{17}$$

3.2.2 Projections on separable two-photon polarization states

Consider a two-photon system in state $|\Psi\rangle$. Then polarization projections consist of two polarization analyzers, PA. Any photon passing such a PA will be detected behind the PA. The PA are placed such that one photon of the systems wents to the first PA and the other photon goes to the second PA. Both photons are assumed to reach the detectors simultaneously, if they have passed the PA. Now, both PA are orientated such that only photons with a certain polarizations can pass and thus be detected. That means that the both PA together can form a projection onto any state of the form

$$|\xi\rangle = p_1\,|p_1\rangle \otimes p_2\,|p_2\rangle \tag{18}$$

where $|p_i\rangle$ corresponds to the single state of photons which can pass the PA number i, and the p_i must obey the normalization condition.

Assume now that the detection is such that only if both photons of a system pass their PA, this is recorded by the detectors. Such an recording event shall be called coincidence. Consider further that an large number N of systems in state $|\Psi\rangle$ is sent to the PA. The number of coincidences C can be calculated as a fraction of N to be

$$C = N \cdot |\,\langle\xi|\Psi\rangle\,|^2 \tag{19}$$

3.2.3 Coincidences in projection measurements

It is important to know what value of C one expects in a certain projection measurement, if one makes the assumption that the system is described by the state vector $|\Psi\rangle$. This motivates the following general calculation of components of the polarization vector. Each projection can be written as a normalized vector

$$|\xi\rangle = \left(ae^{i\alpha}\,|H\rangle + be^{i\beta}\,|V\rangle\right) \otimes \left(ce^{i\gamma}\,|H\rangle + de^{i\delta}\,|V\rangle\right) \tag{20}$$

Now one computes the absolute square of the scalar product of this projection state with any two-photon polarization state $|\Psi\rangle$ one can think of to obtain the quantum-mechanical predictions for the coincidences. However, in this work only states of the following form need to be considered due to experimental circumstances.

1. $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + e^{i\phi}|VV\rangle\right)$
2. $|\Psi\rangle = |HH\rangle$
3. $|\Psi\rangle = |VV\rangle$

The calculations in each of the above cases lead to the following predictions. Set $\epsilon = \frac{C}{N}$.

1. $\epsilon = \frac{(ac-bd)^2}{2} + 2abcd\cos^2(\frac{\Theta}{2})$
2. $\epsilon = (ac)^2$
3. $\epsilon = (bd)^2$

In the first expression, $\Theta := \alpha + \gamma - \beta - \delta - \phi$. To illustrate this expression, consider the projection onto the state $|DD\rangle$, which may be denoted by DD. In this case ($a = b = c = d = \frac{1}{\sqrt{2}}$), and ($\alpha = \gamma = \beta = \delta = 0$). Therefore one obtains $\epsilon = \cos^2\Theta$. From this relations, provided that a state of the first kind was really prepared, one can determine the phase ϕ in the state by only one projection measurement. It appears that for $\phi = 0$ one expects maximum coincidences in the DD projection, and no coincidences at all for $\phi = \pi$.

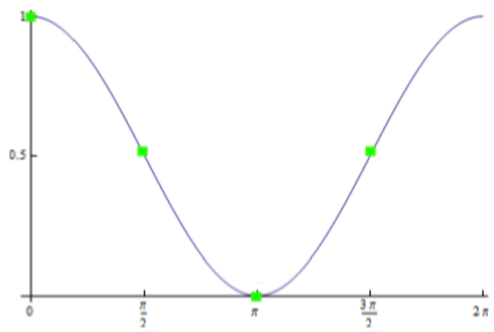


Figure 3: cincidences prediction in DD projection measurement

3.3 Compensation of walk-off effects in uniaxial crystals

3.3.1 Dispersion and birefringence

Consider a photon wavepacket, which travels through a uniaxial crystal. The birefringent nature of the crystal together with dispersion may lead to relative walk-off between photons which differ in energy, polarization and proagation direction.

Dispersion of the group velocity The refractive index n in dielectric media is a rather complicated function of the wavelength, which can be approximated by the Sellmeier equations $n = n(\lambda)$[10][2]. The group velocity is defined[13]

$$v_{group} := \frac{d\omega}{dk} \tag{21}$$

Note that $k = \frac{2\pi}{\lambda}$. Therefore, the group velocity depends on the refractive index. Accordingly, low- and high-energy photons travel at different speed in the medium. This can be illustrated in an suggestive way, if one assumes a wavepacket with finite bandwidth. Due to dispersion, the packet will spread when it moves through the medium (Figure 1).

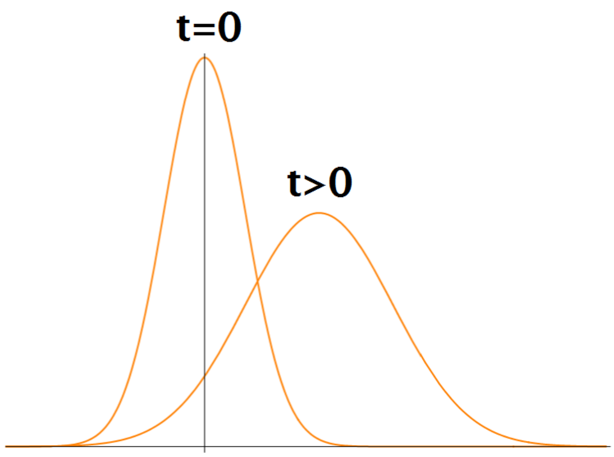


Figure 4: spread of wave packet

[10]note that λ in this context denotes the wavelength in vacuum

The group velocity can also be expressed in terms of the wavelength and the refractive index.

$$\frac{1}{v_{group}} = \frac{1}{c}(1 - \lambda \cdot \frac{dn}{d\lambda}) \tag{22}$$

Birefringence of uniaxial crystals A crystal is called uniaxial crystal if it possesses a particular direction, Z, which is called the optical axis of the crystal. Consider first a light beam which travels through such a crystal. The direction of propagation, $\vec{K}$, together with the optical axis sets up a plane, which is called principal plane. Then, photons which are polarized in this plane, are called extra-ordinary or simply e-polarized. Photons polarized normal to the plane are said to be ordinary or o-polarized (Figure 2).

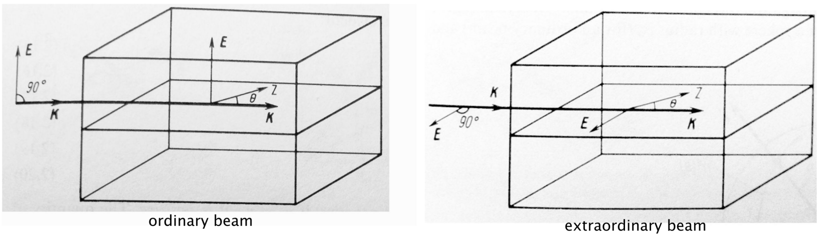


Figure 5: definition of o- and e-polarization[3]

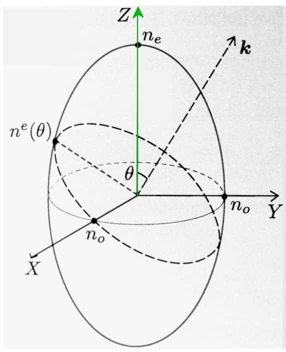


Figure 6: index ellipsoid of uniaxial crystal[3]

The optical axis is special in the following sense. While the refractive index of o-light, n_o, does not depend on the propagation direction, the refractive index of e-light, n^e, does. This effect is called birefringence. It is explained by the anisotropy of the crystal, which leads to a tensorial nature of the refractive index. The dependence of n^e on the angle between optical axis and propagation direction, Θ, can be written

$$\frac{1}{(n^e)^2} = \frac{\sin^2(\Theta)}{{n_e}^2} + \frac{\cos^2(\Theta)}{{n_o}^2} \tag{23}$$

where n^e must not be confused with n_e, which is given by $n_e := n^e(\frac{\pi}{2})$[12]. The birefringence is defined

$$\Delta n := n_e - n_o \tag{24}$$

and the crystal is said to be positive, if $\Delta n > 0$ holds, and negative otherwise. Note that the equation for n^e is of elliptical form. The refractive index of the uniaxial crystal can therefore be represented by an ellipsoid (Figure 3). Note that cuts of the ellipsoid perpendicular to the optical axis are circles, due to the fact that the refractive index for o-light does not depend on the direction of propagation.

3.3.2 Description of walk-off effects

Consider a beam of photons containing a fraction of e- and a fraction of o-polarized photons. If this beam travels through a birefringent crystal, three effects will occur. These are spatial and temporal separation on the one hand, and phase shift on the other hand. In the following, only normal incidence on the crystal surface is considered. This is a loss of generality,since refraction processes are not being accounted for, which necessarily occur in the case of skew incidence. It turns out, however, that in the experiments of this study angles of incidence can be considered small, in the sense that refraction does not significantly affect the walk-off effects.

spatial separation Dependent on the propagation direction, the e-polarized photons are deflected from the o-polarized photons by an angle γ, which is given by

$$\gamma = \pm \arctan[(\frac{n_o}{n_e})^2 \cdot \tan\Theta] \mp \Theta \tag{25}$$

where the upper and lower sign refer to negative and positive crystals, respectively[NIKOGY]. This accumulates to a spatial separation at the output of the crystal, $\Delta l = l \cdot \tan\gamma$. It turns out that this effect is negligible in the performed experiments and was mentioned for completeness.

Temporal separation Because the refractive indices for e- and o-light differ by Δn, e-photon and o-photon travel at different speed in the crystal. This results in a temporal separation between the both. The temporal separation per crystal length l in the case of normal incidence is equal to

$$\frac{\Delta t}{l} = \frac{1}{v_o} - \frac{\sqrt{1 + \tan\gamma}}{v_e} \tag{26}$$

where $v_o = v_{group}(n_o, \lambda)$ and $v_e = v_{group}(n^e, \lambda)$. Temporal separation is a critical effect, since it can lead to decoherence of the entangled two-photon states. This will be discussed in detail in due course.

Phase shift The phase velocity of a wave in the crystal is equal to

$$v_{phase} = \frac{c}{n} \tag{27}$$

and is thus different for o- and e-waves. Therefore, a o-polarized wave will suffer a phase shift $\Delta\phi$ relative to a e-polarized wave, which is found to be

$$\Delta\phi = \frac{2\pi l}{\lambda} \cdot (n_o - n^e \cdot \sqrt{1 + \tan\gamma}) \tag{28}$$

This effect affects only the relative phase in the states. It can actually be used to adjust the phase in the states in a simple way, as explained in

3.3.3 Compensation of temporal separation

Consider two negative uniaxial BBO crystals in the arrangement shown in (Figure 7). Assume that a H-polarized and a V-polarized photon, of the same energy, $\lambda = \lambda_0$, enter the crystals simultaneously. Assume further that both photons undergo SPDC in type I phase matching. The goal is to compute the time interval Δt between the arrival times of the down-converted photon pairs at the output of the second crystal, within a simple model.

Suppose that the SPDC is degenerated in the sense that for the wavelength of the down-converted photons $\lambda = 2\lambda_0$. Additionally, the appearing angles may be considered small, in the sense that all refraction effects are negligible. The photons may all travel along straight lines parallel to the normal incidence direction. In particular, the walk-off angle $\gamma = 0$. We may treat the H-photon first. The H-photon is ordinary polarized in the first crystal and travels therefore at speed $v_o 1 = v_g roup(n_o, \lambda_0)$. It is extraordinary polarized in the second crystal and travels at $v_e 1 = v_g roup(n_e, \lambda_0)$. After down-conversion has occured, the down-converted V-pair travels at

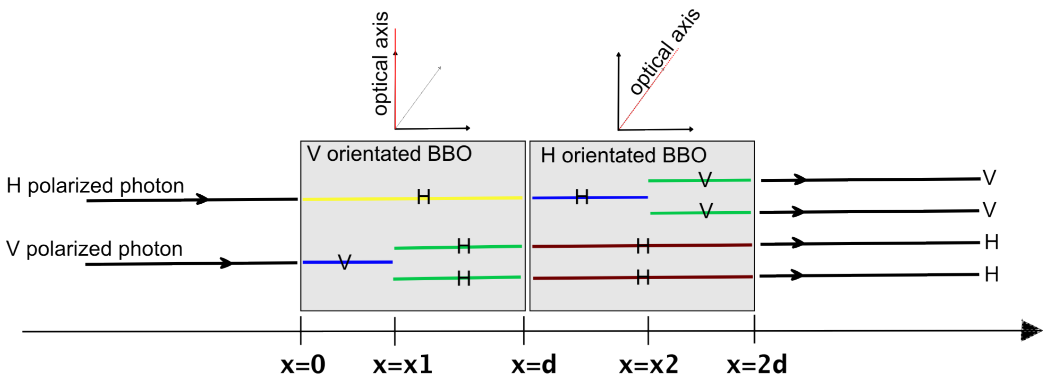


Figure 7: temporal separation in typ I down-conversion

$v_o 2 = v_g roup(n_o, 2\lambda_0)$. Analogue reasoning goes for the V-photon. Using the illustration, one obtains for the time interval Δt the following expression.

$$\Delta t = d \cdot (\frac{1}{v_{o1}} + \frac{1}{v_{o2}} - \frac{1}{v_{e1}} - \frac{1}{v_{e2}}) + (x_2 - x_1) \cdot (\frac{1}{v_{e1}} - \frac{1}{v_{02}}) \tag{29}$$

It may be assumed that SPDC occurs equally likely on each point in the crystal. Then one finds the average time interval $\bar{\Delta t}$ to be

$$\bar{\Delta t} = \epsilon \cdot d \tag{30}$$

where $\epsilon := \frac{1}{v_{o1}} - \frac{1}{v_{e2}}$. This is not zero in general. Therefore the temporal walk-off leads to an average arrival delay between H-pairs and V-pairs. This can be compensated in a straightforward manner. One may place another uniaxial crystal into the path of the photons, such that the temporal separation in this so called compensation crystal is equal to $\bar{\Delta t}_c omp = \bar{\Delta t}$. To avoid SPDC in the compensation crystal, one has to ensure that the phases are not matched. This is achieved in this example if one uses negative uniaxial crystals such as YVO4. We will come back to this compensation problem in the experimental part.

4 Presentation of the Experiments

The experimental work can be devided in three stages. First, in a Typ I SPDC setup, entangled two-photon states of the form $|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|HH\rangle + e^{i\phi} |VV\rangle\right)$ are prepared. Second, several measurements are performed to prove the actual production of entanglement. Third, it is shown that decoherence of the entangled state can be obtained by temporal separation.

4.1 Preparation of maximally entangled states

For the experiments of this study, a readily implemented setup was used, which is phase-matched for TYP I SPDC. This subsection shows how the components of the setup contribute to the preparation of maximally entangled states.

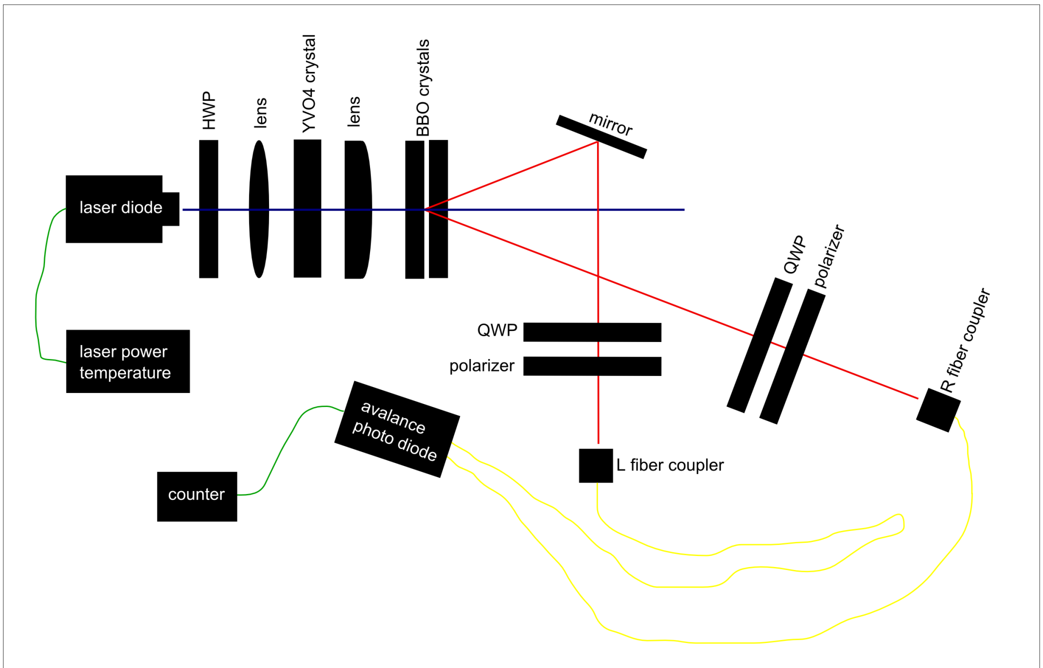


Figure 8: experimental setup

4.1.1 SPDC source

Laser diode As a source of polarized photons, a laser diode is used which produces H-polarized[11] photons at wavelength $\lambda_0 = 405nm$ with a bandwidth of 3nm. The coherence length l_c of this laser diode was not measured directly but is only of order of a few hundred μm. Thus, although it is not a pulsed laser, the diode can be considered as pulsed to a good approximation. Lenses are used to properly collimate the beam.

BBO crystals As medium for SPDC to occur, two BBO crystals of thickness $d = 0.7mm$ are exposed to the laser beam such that one optical axis of the crystals is orientated in horizontal(H) direction and the other axis is

[11] per definition

orientated vertically(V). Due to phase-matching conditions, SPDC can now only occur in the V-orientated crystal. Therefore, one only produces pairs of H-polarized photons.

Half-wave plate A half-wave plate, HWP, is then placed directly behind the aperture of the laser such that it rotates the polarization of the beam from H to D. Therefore, SPDC will now occur in both crstals with equal probability, because the beam now behaves in the crystals like a mixture of an equal number of H- and V-polarized photons.

Spatial distribution of the down-conversion photons The implementation is such that the downconversion photons emerge with equal energy. Due to birefringence of the BBO crystals and phase-matching conditions, the down-conversion photons come out on cones with circular cross-section.

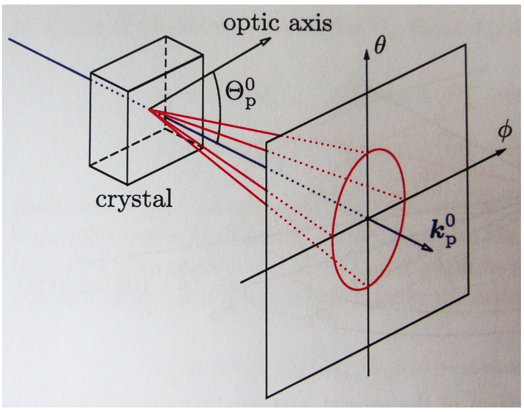


Figure 9: Cone of down-converted photons[3]

The cones are widened up a bit due to the finite bandwidth of the pump photons. There will appear two cones, one from each crystal, which can made overlap by adjusting the relative orientation of the crystals. Note that due to momentum conservation, the two photons from a certain decay come out on opposite points on the cones.

Collection of correlated photon pairs To collect correlated photon pairs, one has to focus two fiber couplers onto points on the cones, which are directly opposite. Otherwise, two simultaneously collected photons would not be correlated, since they emerged from different decays. The adjustment of the couplers has to be done very precisely. It is very important that both couplers "see" the same amount of down-conversion light from both crystals. A dysbalance will lead to reduction of the correlations.

Compensation The idea of this experiment is that entanglement arises from the indistinguishability of HH and VV pairs prior to the polarization measurement. If it is not possible to predict a priori, from which crystal a certain photon pair has emerged, for example by measuring arrival times of the pairs on the crystal output, then a state of the form $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + e^{i\phi}|VV\rangle\right)$ is always prepared. Until now, the produced two-photon polarization states are not in general entangled. Birefringence and dispersion give rise to the effect of temporal separation. If the temporal separation is larger than the coherence time of the laser diode, there would be no phase relation between the photon pairs and this leads to distinguishability and therefore destroys the entanglement of the state. To ensure, that the photon pairs keep being indistinguishable, the concept of compensation has to be applied. With the Sellmeier equations for BBO, one obtains the average temporal separation at the crystal output $\bar{\Delta t} = 5 \cdot 10^{-}13s$. If one assumes a coherence length of the laser diode of about $100\mu m$, this would imply a coherence time of $t_c = \frac{l_c}{c} = 10^{-}12s$. The temporal separation is of order of the coherence length and should therefore be compensated. The compensation shall be achieved by an additional compensation crystal. YVO4 was chosen for several reasons. It exhibits strong birefringence, and needs therefore not to be very thick. Since it is positive uniaxial, there will not occur SPDC in the crystal. The required thickness is obtained to be $d_{comp} = 0.34mm$. Since $100\mu m$ is considered as a lower bound on the coherence length, a YVO4 crystal of thickness $0.20mm$ is used. If it is possible to prepare maximally entangled states in this setup, this would imply a higher lower bound on the coherence length. This question will be answered in due course.

Phase adjustment In this work, the state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ shall be produced. The total relative phase shifts between H- and V-light in the optical components is not zero in general as it must to obtain the state $|\Psi^+\rangle$. Nevertheless, it can be adjusted to zero by tilting the compensation crystal by a small amount.The length of the crystal in the beam direction increases and therefore the relative phase changes.

4.1.2 Concepts of polarization measurement

Coincidence counts It was already mentioned that two fiber couplers are set on opposite points on the cones. The fibers tranfer the signals of incoming photons to an avalanche photo diode. The signals are subsequently lead to a counter. The counter returns the single rates on each PA. All measurements in this study are based on coincidence counts. Coincidence is defined in this experiment by a time interval $\delta\tau = 20ns$. If both detectors are hit by photons during this time interval, the counter treats this event as a coincidence. Since the time interval is finite, the measurement may be disturbed by accidental coincidences. A estimate on the influence of such accidental $c_a$counts is needed to understand the significance of the obtained results. This estimate is given in terms of the single count rates (n_1, n_2)as follows.

$$c_a = \delta\tau \cdot n_1 \cdot n_2 \tag{31}$$

In the concerned experiments, the coincidence rates are about $1000/sec$ and the single counts are about 30000/sec. Therefore one obtains $c_a \approx 18$ which is negligible.

Projection measurements The PA in the performed experiments consist of a quarter-wave plate, QWP, and a polarizer P. A quarter wave plate can transform linear polarization into circular polarization and vice-versa. To project in the right way on the desired states, QWP and P must be positioned like it is illustrated in (figure). In the following table, settings of the devices for the needed projection measurements are presented. To be consistent, the axis of the QWP and the defined H-direction are set to angle zero.

projection	QWP	P
H	0	0
V	0	90
D	45	45
R	45	90
L	0	90

Figure 10: important polarization projections

4.2 Experimental implementation of polarization measurements

4.2.1 Correlations measurement

This experiment is performed to show the existence of non-classical correlations between the two photons of a pair. In the first part of the experiment, both polarizers are set to H. One polarizer is then subsequently rotated in steps of ten degrees, and the coincidences are recorded for each setting. That means that the analysis is carried out in the HV basis. A sinusoidal dependence of the coincidences is observed, what proves the existence of correlations between the photons of a pair. A measure for the quality of these correlations is provided by the visibility, which in this case is defined as the contrast of the sinusoidal curve

$$v = \frac{n_{max} - n_{min}}{n_{max} + n_{min}} \tag{32}$$

The expression for the uncertainty in the visibility due to Poisson fluctuations reads

$$\Delta v^2 = \frac{4n_1 n_e}{(n_1 + n_2)^3} \tag{33}$$

For the measurement, one obtains $v = (94.2 \pm 0.7)$ and therefore very strong correlations.That shows actually, that both photons of a pair always have the same polarization. However, this does not prove any non-classical behaviour, since the same result would have been obtained, if the photon pairs were in the mixed state $\frac{1}{2}\left(|HH\rangle\langle HH| + |VV\langle VV|\right)$.
Therefore, the measurement was repeated, but this time one polarizer was held fixed at D, therefore the measurement was now performed in $D\bar{D}$ basis. Again, the same behaviour was observed, with comparable visibility, $v = (98.5 \pm 0.3)\%$. Note that for the mixture suggested above, one would have recorded a flat curve. This invariance with respect to basis change reflects the existence of non-classical correlations in the state of the photon pair. The measured curves are displayed below. Note that the integration time was 2.0sec.

It is possible to extract from the diagram that the relative phase ϕ was not equal to zero in this measurement, if one recalls the predictions for coincidences obtained in (2.2). Analysation angle equal to $\frac{\pi}{4}$ corresponds to a DD projection in the blue case. If the relative phase is equal to zero one expects maximum coincidences for this projection. But the result shows nearly a minimum for that setting. With the expressions derived one finds approximately $\phi \approx 0.83\pi$.

4.2.2 Violation of the CHSH inequality

The CHSH inequality This deduction follows the treatment in [4]. Assume an entangled two-photon polarization state which shall be measured by projection measurements. Assume further that each of the PA can measure on "its" photon two physical quantities (A, B) and (C, D), respectively, which can take values (± 1). Consider the quantity $S = AC + BC + BD - AD$. Then, $S = \pm 2$. Prior to the measurements, the bipartite system may be in

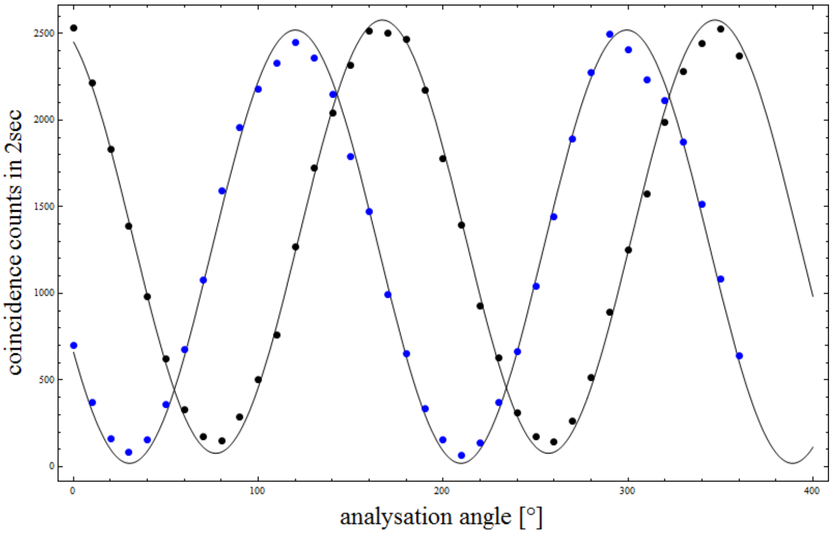


Figure 11: correlations measurement in HV(black) and $D\bar{D}$(blue)

a state where $A = a, B = b, C = c, D = d$ with probability $p(a, b, c, d)$. This is the principle of reality, and the $p(a, b, c, d)$ can be associated with hidden variables. Furthermore, it is assumed that both PA measure randomly one of the two quantities, without influencing each other, so that the principle of locality is enforced as well. Under these assumptions, the CHSH inequality is obtained by simple considerations on mean values E of the contributions to S. CHSH showed that within any LHV theory,

$$|S| = |E_{AC} + E_{BC} + E_{BD} - E_{AD}| \leq 2 \tag{34}$$

holds. What is more, CHSH were able to predict from the previously introduced quantum-mechanical principles, that there exist measurements, which violate the CHSH inequality by a factor of $\sqrt{2}$, if performed on maximally entangled states of such kind as the $|\Psi^+\rangle$.

Measurement settings The four quantities are single photon polarization bases, which are shifted by $\pi 8$. The value $+1$ corresponds to cases in which coincidences appear, and -1 corresponds to no coincidences. The different measurements of the left and right PA are presented below in (figure 12).

Results The measurement was performed after the phase was adjusted. The measurement results were displayed in (figure 13)..

The mean values to be computed are all of the same form. Consider for example the case E_{AC}.

$$E_{AC} = \frac{n_{0/22.5} - n_{90/22.5} - n_{0/112.5} + n_{90/112.5}}{n_{0/22.5} + n_{90/22.5} + n_{0/112.5} + n_{90/112.5}} \tag{35}$$

Very important in this case is a discussion on uncertainties. Again, it is assumed that the coincidence rates are Poisson distributed. One obtains

$$(\Delta S)^2 = (\Delta E_{AC})^2 + (\Delta E_{BC})^2 + (\Delta E_{AD})^2 + (\Delta E_{BD})^2 \tag{36}$$

$$(\Delta E_{AC})^2 = \frac{1 - E_{AC}^2}{n_{0/22.5} + n_{90/22.5} + n_{0/112.5} + n_{90/112.5}} \tag{37}$$

The same goes for the other mean value uncertainties.

The evaluation of the measurement results yields $S = 2.54 \pm 0.04$. That means that the CHSH inequality was violated by well above thirteen standard deviations. This serves as a proof that nature cannot be completely described on grounds of reality and locality. The result of the experiment provides strong evidence that qauntum theory describes nature very well and is a dead end for all LHV theories.

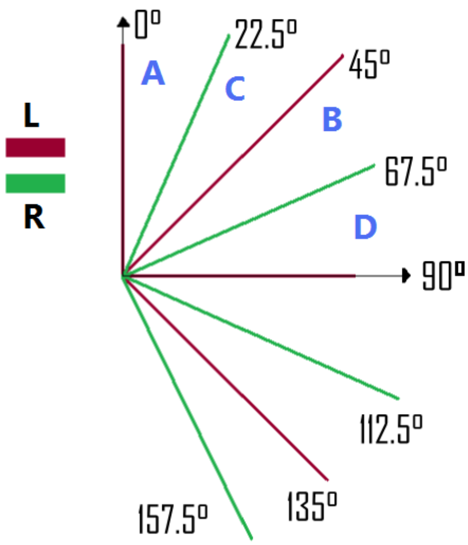


Figure 12: CHSH settings

4.2.3 State Tomography

The state tomographies in this work are based on the method proposed in[14]. The 16 projection measurements set up a polarization vector $|P\rangle$. It

R\L	0°	45°	90°	135°
22.5°	2192	388	2102	433
67.5°	584	2183	483	2058
112.5°	455	2100	2099	428
167.5°	2104	466	510	2025

Figure 13: CHSH results

was possible in the experiments to prepare a $|\Psi^+\rangle$ state with high accuracy. Consider first the reconstruction of the density operator.

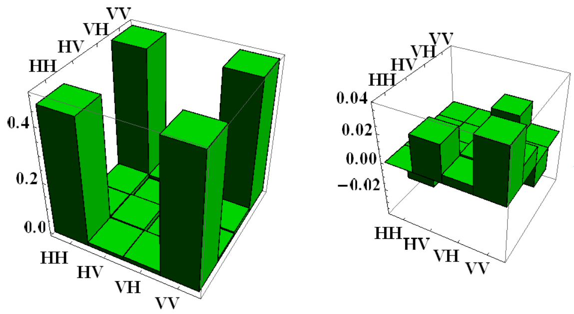


Figure 14: Tomography of Psi+ state

The fidelity in this case is equal to $F = 0.98$. Furthermore, $Tr(\rho^2) = 0.96$.

The Peres-Horodecki could have also been checked in this work.It was omitted for several reasons. First of all, the reconstruction of the density operator is not necessarily positive semi-definite. Therefore, it could have negative eigenvalues. This diminishes the meaning of a probable violation of the PPT criterion. This problem could be solved by performing a maximum likelihood approximation. This is an algorithm which fits reconstructed density operators on physical operators if needed. But this approximation was not part of this study. It is emphasized, however, that the PPT criterion provides an excellent test of entanglement, since it is necessary and sufficient in the bipartite case.

4.3 Induced decoherence by temporal separation

Setting up from the discussion on compensation, one can design an experiment to show, that the coherence decreases with increasing temporal separation of the H- and V-pairs. Starting from the phase adjusted $|\Psi^+\rangle$ state, first the compensation crystal is rotated by an angle of ninety degrees. One then expects decreasing of the coherences, because the compensation effect now goes the other way round, leading rather to an amplification of the temporal separation caused by the BBO crystals. This is because a rotation by ninety degrees exchanges the roles of H- and V- light in the compensation crystal.

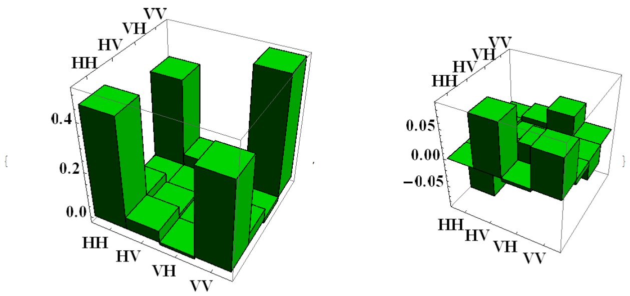


Figure 15: Tomography after rotation of the YVO4 crystal

Indeed a small decreasing of the coherences is observed. The fidelity has droppes to $F = 0.87$. $Tr(\rho^2) = 0.81$. As a next step, the $0.20mm$ compensation crystal is replaced by a $0.60mm$, otherwise identical YVO4 crystal in original compensation orientation. A further diminishing of the coherences is expected, since the separation is now over-compensated by a factor of two.

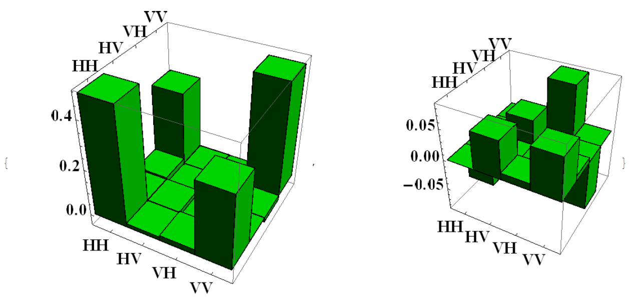


Figure 16: Tomography after exchange of the 0.20 YVO with a 0.60 YVO in compensation orientation

As predicted, a significant reduction of the coherences appears. The fidelity is only $F = 0.80$, and $Tr(\rho^2) = 0.70$, which shows a non-negligible amount of mixture in the state. To fully transform the state into a decoherent mixture of H- and V-pairs the $0.60mm$ is rotated in a last measurement by ninety degrees. Now, significant evidence of a mixed state is expected, since the previous results indicate that the coherence length of the laser diode does not significantly exceed the scale of several hundred microns. Really, the state tomography showed a very good result.

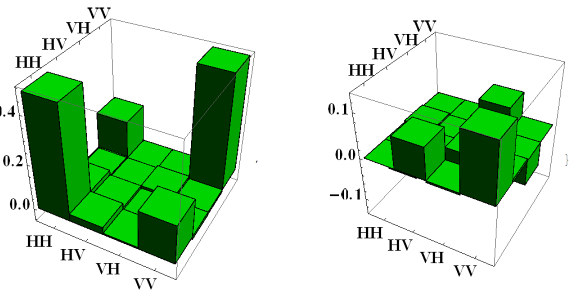


Figure 17: Decompensation has lead to decoherence

The coherence has almost vanished. The fidelity is only $F = 0.66$, and $Tr(\rho^2) = 0.60$. Therefore, it was possible to show, that the quantum correlations can be contiuously diminished, by controlled change of the compensation. Now, from the last result one can conclude an upper bound on the coherence length of the laser diode. The separation due to the BBO crystals, which equals $\bar{\Delta t} = 5 \cdot 10^-13s$, is amplified by additional separation in $0.60mm$ YVO4. The additional separation is roundabout $8.8 \cdot 10^-13s$, so that the total separation is $1.4 \cdot 10^-12s$. It follows that the coherence length is certainly less than 420microns. The coherence length is therefore bounded, within this simple model, between 100 and 400 microns.

5 Conclusion

In this study, entangled two-photon polarization states were efficiently produced in a special configuration of Typ I SPDC. The efficient production is important with respect to communication. It is necessary in communication to transmit information at a high rate, so one of the cornerstones of quantum information is the search for very efficient sources, like SPDC has proved to be one of them. The detection of entangled states was a further part of this work. Several entanglement criteria and their application in experiment were explained and used to test experimentally the quality of the prepared states. It was possible to prove non-classical correlations in the entangled states. The CHSH inequality was violated. This is a strong evidence of non-locality and/or non-reality of nature. By reconstruction of the density operator, additional proof of the production of the desired entangled state was provided. Recall, however, that the used method of state tomography was not perfectly rigorous, since the obtained density operators are not positiv semidefinite per construction. It shown to be possible to change the relative phase by controlling the relative phase shift between photons of different polarization in birefringent crystals. Last but not least, decoherence of the entangled state could be successfully induced by temporal separation. It was shown, starting from a maximally entangled state, that the coherence drops with increasing temporal separation. This result underlined the importance of precise compensation in such setups. To summarize the course of the study in one sentence, a deep insight in the production and detection of entangled states was obtained as well as knowledge about the phenomenon of decoherence.

6 Appendix

6.1 Error in Poisson distributed quantities

Error calculations The coincidence counts measured in course of this study can be considered poisson distributed. That implies, that one cannot predict with certainty the coincidence counts in a given experiments, even if all setup configurations are perfectly specified. The counts are uncertain by an amount yet to be specified.
To draw meaningful conclusions from any performed experiment, error calculations are needed to reveal the degree of uncertainty in the value of any physical quantity deduced from the measured results. In this study, for example, it is intended to show a significant violation of the CHSH inequality $S \leq 2$. Assume that one measures the result $S = 2.15 > 2$ with uncertainty $\Delta S = \pm 0.25$. In this case the validity of the CHSH inequality basically can not be abandoned. Significance of the result in this case would require an uncertainty $\Delta S < 0.15$.
The conventional procedure to find the uncertainty of any quantity Q deduced form measurement results q_i, is called error propagation. One determines the dependence of Q on the q_i, and specifies the uncertainty Δq_i in the values q_i. Thus the uncertainty of Q can be written

$$(\Delta Q)^2 = \sum_k \left(\frac{\partial Q}{\partial q_k} \right)^2 \cdot (\Delta q_k)^2 \tag{38}$$

Application to Poisson distribution In course of this study, the measurements are based on coincidence counts n, which can be considered Poisson distributed. In this case, one specifies the uncertainty Δn of the coincidence counts to be

$$\Delta n_i = \sqrt{n_i}. \tag{39}$$

With the general expression for the uncertainty of a quantity Q, this uncertainty becomes

$$(\Delta Q)^2 = \sum_k \left(\frac{\partial Q}{\partial q_k} \right)^2 \cdot \Delta q_k \tag{40}$$

This relation is based to all error considerations in course of this study.

6.2 Optical properties of the used uniaxial crystals

6.2.1 YVO4 crystals

Crystal class positive uniaxial

Sellmeier equations

1. $n_o^2 = 3.77834 + \frac{0.069736}{\lambda^2 - 0.04724} - 0.0108133\lambda^2$
2. $n_e^2 = 4.59909 + \frac{0.110534}{\lambda^2 - 0.04813} - 0.0122676^2$

Refractive index at used wavelength

1. $n_o(405nm) = 2.09$
2. $n_e(405nm) = 2.36$

6.2.2 BBO crystals

Crystal class negative uniaxial

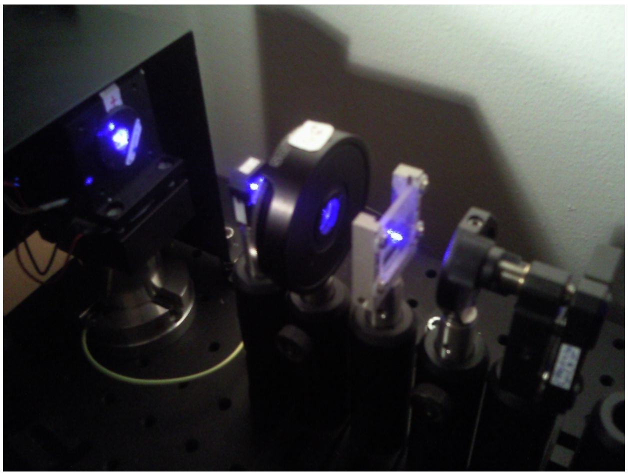
Sellmeier equations

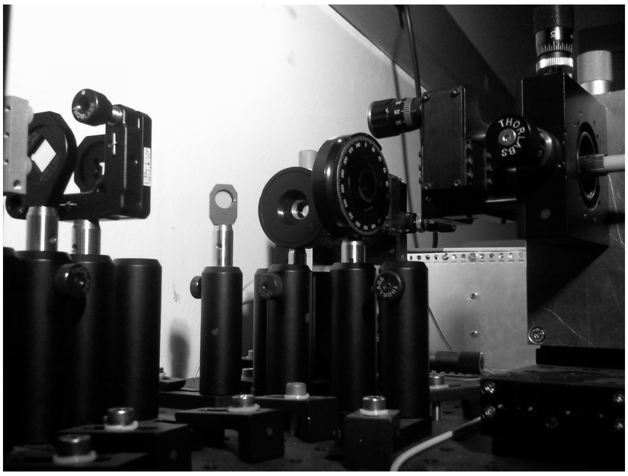
1. $n_o^2 = 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2$
2. $n_e^2 = 2.3753 + \frac{00.01224}{\lambda^2 - 0.01667} - 0.01516^2$

Refractive index at used wavelength

1. $n_o(405nm) = 1.69$
2. $n_e(405nm) = 1.57$
3. $n_e(810nm) = 1.66$
4. $n_e(810nm) = 1.54$

6.3 Pictures of the setup





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Selbstständigkeitserklärung

Hiermit versichere ich, dass ich die vorlegende Arbeit selbständig und nur mithilfe der angegebenen Quellen verfasst habe.

Mnchen, den 18.Mai 2009

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